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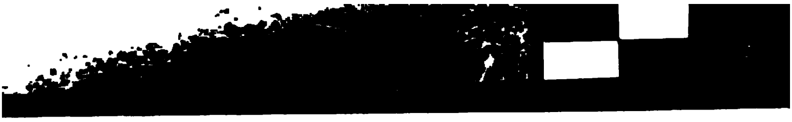

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NOTES ON A COURSE

OF

Lectures in Kinematics

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PREFATORY NOTE.

In these notes no claim to originality is made. A large part of the matter is taken, with only slight variation, from Prof. A. B. W. KENNEDY'S excellent book, "The Mechanics of Machinery." The object of issuing them is simply to bring together in a convenient form for use by the student the substance of that which it has been deemed advisable to give in the lectures.

A. W. S.

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NOTES ON LECTURES IN KINEMATICS.

The object of the present course is to help students to become machine designers.

The machine designer should understand clearly what may be called "The Mechanics of Machines." This subject differs from the general subject of Mechanics, in that it is concerned solely with the study of moving bodies, whose motion is *constrained*.

Mechanics of Machines may have three subdivisions as follows :

(1.) Kinematics of Machines.

(2.) Statics of Machines.

(3.) Kinetics, or Dynamics of Machines.

In (1) motions are considered independently of the forces that produce them.

In (2) the transmission of force by bodies that do not move, is considered ; *i. e.*, it is the study of forces in equilibrium.

In (3) forces in moving parts are studied ; and since it is a study of the laws that govern forces in motion, it is therefore the science of "work."

Let this distinction be illustrated. Take the case of the ordinary steam engine. Force may be disregarded, and by the laws of Kinematics the relative velocities of the piston, crank, connecting rod, and in fact of any of the moving parts during an entire revolution may be determined. But the engine *bed* is a stationary part, and yet it transmits the reactions of the forces that the moving parts transmit, and therefore the study of the stresses that the bed has to withstand, is the study of a problem in *Statics*. If on the other hand we consider the forces in the moving parts, as the connecting rod for instance, we have a problem in *Dynamics*.

The Kinematic problems are the simplest and therefore they are to be considered first in order.

Since we are to deal with the study and design of machines, it is fitting to get a clear idea of what constitutes a machine at *the very start*. I shall therefore now give and discuss *Professor Kennedy's* definition of a machine.

"*A machine is a combination of resistant bodies, whose relative motions are eompletely constrained, and which may be utilized to transform the natural energies at our disposal, into any special form of work.*"

In the definition a machine is said to be a *combination.* No single body can constitute a machine. Take for instance a lever; all of its capability to act as a machine, depends upon the existence of a fulcrum; without such a fulcrum, it is a mere bar, incapable of the slightest mechanical use; whereas with such a fulcrum, properly designed and constructed, it becomes one of the most important combinations with which we have to deal. Or take the "wheel and axle," (which is one body, although usually built up of two or more,) it is utterly useless mechanically till bearings are provided to support the axle, and the bearings joined to each other. Thus a counter shaft, or a grindstone, or a lathe-spindle, each of which is a wheel and axle, are useless till bearings are supplied and joined together.

It will be seen that the adjective *resistant* is applied in the definition, to the bodies that go to make up a machine. A large proportion of these bodies are rigid; rigidity, however, is not an essential condition. Springs of steel, or brass or rubber, are sometimes used, and they yield to the forces applied to them; in fact they are used simply because they do yield. Fluids are also often used. Let us consider a case. Let *A* and *B* be cylindrical vessels in communication with each other. Let *C* and *D*, (Fig. 1,) be tight pistons working in these vessels. Let the space below these pistons be filled with water. If either piston be depressed, its motion will be communicated to the other by means of the water. It will be seen then that the water in this case is in the fullest sense, a part of the machine, since it transmits force in motion. But water is not rigid, as we should quickly observe if we were to try to use it for the transmission of force by means of tension. A rigid steel or iron bar would serve to transmit the motion of one piston to another, as in a tandem engine, but in many cases, as for instance, the hydraulic press, the fluid serves the purpose with far greater convenience. Flexible bands, belts and ropes are also sometimes used in machines, e. g., a rope over a pulley serves to transform muscular energy into the work of lifting a weight; but if an attempt is made to trans-

mit motion by reversing the direction of the force, the rope fails from its lack of rigidity.

The question arises, what is the essential condition that a body must fulfill in order that it may become available for a machine part, if rigidity is not. It is that it shall present, or be made to present, a suitable molecular resistance to change of form or volume, a quality that is expressed by the term resistant. The reason, therefore, for the use of this term will be clear.

The relative motions of the resistant bodies are said in the definition to be *constrained.* This point requires more extended consideration, because it is the chief characteristic that distinguishes the problems in Mechanics of Machines, with which we are to deal, from the problems in the general subject of Mechanics.

In the general case the bodies whose motions come into consideration are FREE; in the cases that will come under our notice the motion will be CONSTRAINED.

When a body is free, the direction of its motion is determined by the direction of the force producing the motion, and it may be changed at any instant by changing the direction of the moving force, or by the introduction of disturbing forces.

On the other hand, when the motion is constrained, its direction is entirely independent of the direction of the force that produces the motion, and it may not be changed either by introducing other forces, or by the changing the direction of the actuating force. The motion of the free body for any given instant, is only known when every force that shall act on it during that instant, intentionally or otherwise, is known. If the machine problem had to be treated in that way, every knock or jar, even a hand pressure, would need to be considered; in fact it would become an entirely impossible problem.

In the case of constrained motion, however, to which machines fortunately belong, the direction of motion is at every instant completely determined by the construction. The only alternative, is motion in the right direction, or no motion at all. As long as the machine is in motion, all its parts are compelled *to move* in prearranged paths, and no force can *change them,* unless it is sufficient to distort or destroy *machine.* Summing up then, we may say:

As long as a machine remains uninjured, all its motions as to their direction, are entirely independent of the direction or intensity of the forces causing them.

We say that the direction of motion is independent of the direction of the forces causing it; but in machines, as in the rest of the Universe, motion is a resultant of all the forces acting, and we must virtually use the same method of getting rid of disturbing forces here, as elsewhere; *i. e.*, balance them. In general, the resultant of all the forces acting, is oblique to the direction of motion desired; and it may be resolved into components, one coinciding with the direction of motion desired, and the other at right angles to it. If the latter be balanced, all is done that is necessary to insure motion in the right direction.

For illustration, let *A*, (Fig. 2,) be a body that is desired to move over the path *AB*; and let *AC* represent the resultant of all the forces acting on the body. Let *AC* be resolved into components along *AB*, and at right angles to it; *AD* and *AE* are the component forces. If now *AE* be balanced and *AD* left free to act we shall have the required direction of motion.

To apply this to a free body, suppose *A*, (Fig. 3,) to be a free body that has been elevated above the surface of the earth, and its desired motion is toward the centre of the earth, along the line *AB*; but wind pressure tends to make it depart from this path in falling, and to follow *AC*. In order that it may follow the desired path, it is necessary to meet this wind pressure from instant to instant by a force equal and opposed. This, while theoretically possible, is not practically so.

In machinery a different plan is employed. The disturbing forces are balanced by connecting the moving bodies in such a way that departure from the desired motion can only occur if the form of the connection be changed; and the connection itself is made of such material and dimensions that it will not yield easily to the forces that act upon it. *The disturbing forces are, therefore, balanced by the resistance of the material of the machine to change of form.* This molecular resistance is called *stress*. Therefore it may be said that *the constrained motion characteristic of bodies in machines is obtained by so connecting them that disturbing forces, as cur, are balanced by stresses in the bodies themselves.*

It is often said that the parts of a machine have their motion constrained by the *form* of the connection between them, and this is true to a certain extent.

Thus, if we wish the motion of a body to be rotation about an axis, we make some part of the body cylindrical, the axis of the cylindrical part being coincident with the axis about which it is desired to have rotation occur, and cause this cylindrical part to work in accurately fitted bearings, rings or collars being provided to prevent endlong motion. Clearly now the motion is determined by the form of the connection. But suppose the bearings to be made of some material like india rubber, that yields easily to forces; the form alone could not completely constrain the motion. It is not then sufficient to make the connection of proper form; it must also be made of proper material. And so the constrainment must always be ultimately referred to the molecular resistance of the material, and not to the form of connection.

We cannot, of course, find materials that will resist *all* possible forces that may be applied to them, or we could make an unbreakable machine. But we can easily find materials that will resist all the disturbing forces that come ordinarily upon machines. True, a machine part may sometimes be distorted or broken, but this must be because of some very unusual stress, or else because the part was not properly designed. It is the province of the machine designer to see that these do not occur, and we need not consider them as yet. We shall, therefore, in the present study, assume that all motions are completely constrained; the direction of motion will be determined by the form of connection; and all disturbing forces will be assumed to be balanced by stresses in the material of the bodies themselves.

The possibility of making this assumption greatly simplifies the problem, because motions occurring in a machine can be considered quite independently of the forces producing them. The paths in which different points move, as well as their relative velocities, may be determined upon purely geometric principles.

We have left now only the last clause of our definition for consideration, which says that the object of a machi[illegible] *transformation of* natural energies into special wor[illegible] *chine is often* spoken of as transmitting and mod[illegible]

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and while this is true, it is not by any means a characteristic of a machine. A bridge does the same; or a roof; or, in fact, any structure. A machine was never made solely to modify force. *Motion* is an essential of a machine; and force in motion, is work, or energy. The special work to be done may be grinding a lathe tool, or cutting a thread, or crushing stone, or any one of a thousand things. The natural energy may be muscular energy, or gravitation energy, or electrical energy, or, as in most cases, heat energy, and the function of a machine is to adapt one to the other. It is precisely this which distinguishes a machine from a mere structure. The latter modifies force only, not energy.

We have now defined and discussed a perfect machine. Some combinations that are called machines depart from the conditions given; as, for instance, "chain hoist," in which the load is only constrained in the vertical direction; but just so far as the conditions of the definition are unfulfilled, the combination departs from a perfect machine.

A certain class, as safety valves, stop valves, perform their function just as fully when at rest as when in motion. Whether this be called machines or not is immaterial. At least they act as machines when in motion, and the problems may be solved by just the same methods as in the case of perfect machines.

It has been seen that only *constrained* motion is dealt with in "Mechanics of Machines."

There is no impossibility about constraining any kind of motion whatever, no matter how complex. The immense majority of motions occurring in machinery, however, fall in a few special cases, and this makes their treatment comparatively simple. We need consider but three of these special cases of constrained motion:

1st. Plane motion.

2d. Spheric motion.

3d. Screw, or Helical motion.

When a body moves so that any one section of it remains in the same plane, its motion is said to be plane motion. A very large proportion of all the motions occurring in machinery are of this class.

In considering problems in plane motion we can introduce the very important simplification, that instead of dealing

the bodies themselves we may treat each body as if it were merely a section of itself, *i. e.*, a plane figure. This section moves in its plane and all its motions may be completely represented on a plane, as a sheet of drawing paper.

There are two forms of plane motion that are of special importance to us. The first is *simple rotation*. When a body rotates about an axis every section of it at right angles to the axis remains in its plane and rotates about a point which is the intersection of the axis with the plane.

The second form of plane motion is that in which all points of a body move in parallel straight lines, *i. e.*, the body moves parallel to itself. This motion is called *motion of translation*. If in the case of rotation we consider the radius to continually increase, the curved path of any point in the rotating body continually becomes flatter, *i. e.*, approaches a straight line, and when the axis is removed to an infinite distance the curve actually becomes a straight line, and instead of having a motion of rotation, it has one of translation. It is obvious then, that translation may be considered the special case of rotation about a point at infinite distance, and, as we shall see later, it will greatly simplify some kinematic problems to so consider it.

If a point A move in any path whatever, but in such a way that it maintains a constant distance from some fixed point B, then A moves in the surface of a sphere of which B is the centre. If all the points of a body move like A, the body has what may be called *spheric motion*. A cone A, rolling on a plane B, or upon another cone C, the apex of A being fixed, is an example of spheric motion.

The third special case of motion that we are to consider is helical or screw motion.

When the motion of a point is such that it may be resolved into a rotation about an axis, and a translation parallel to that axis, the point describes a helix or screw line. It is possible for a body to so move that all of its points shall describe helices about an axis, and such motion is called helical or screw motion.

Each point in the body remains at a constant distance from the axis, and moves through the same angle of rotation, and through the same distance parallel to the axis. The amount

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of translation for a complete rotation is called the *pitch* of the helix. Helical motion stands in a very simple relation to plane motion, in which it resolves itself in two limiting cases: 1st. If we make pitch = 0, obviously the screw motion becomes simple rotation. 2d. If we make pitch = ∞, the screw motion becomes simple translation.

These, then, are the three special cases of motion. More complex motions do occur in machinery, but so rarely that we do not need to consider them in this general discussion.

It is seen that the distance between a body and some other body undergoes alteration, and it is said that the first body changes position, or *moves* relatively to the second.

RELATIVE MOTION.

We know that all bodies around us are continually changing their positions in space; yet we are not able to realize or to measure their absolute motion.

The choice of this second body, the standard, is arbitary. In general it has no motion that is visible with reference to any other body. Thus, ordinarily we say that a body moves or is at rest, according as it moves or is at rest relatively to the earth. But sometimes a body that is itself moving relatively to the earth is selected as a standard; as, for instance, a vessel or a railway train, and those bodies that do not move relatively to them are said to be at rest, no matter what motion they have relatively to the earth. It is easy to suppose that a body on a train has a motion in the opposite direction from that of the train, and equal in amount. The body would have no motion relatively to the earth, or if it be at rest in the train, then it has no motion relatively to the train, although moving relatively to the earth. In the first case the body is stationary relatively to the earth, and in the second, is stationary relatively to the train.

It is necessary to get a clear idea of what is meant by the term *stationary*. In the first supposed case, the body shared the motion of the earth; in the second, of the train. In both cases it shared the motion of the standard by which its motion was measured. When we say that a body is at rest, we assume that it shares all the motions, *i. e.*, the absolute motions of the standard. The result is the same as if the standard were *at rest*.

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It is obvious that if a body be at rest relatively to another, it may be considered a part of that other. The two might be connected rigidly without any change in the conditions.

In machinery generally motion is measured relatively to the frame of the machine which is at rest relatively to the earth. Frequently, however, it is necessary to measure the relative motion of two parts of the machine that are both moving relatively to the frame.

It has been seen that if a body has the same motion as the standard, it is at rest. Two bodies that have the same motion as the standard are then at rest relatively to each other, *i. e.*, they can have no relative motion.

It may, therefore, be stated in general terms that if two bodies have no relative motion, they must have the same motion relatively to every other body. Also, that if two bodies have the same motion relatively to another body, they have no relative motion. Also, the relative motion of two bodies is not affected by any motion they may have in common. For whatever the common motion may be it leaves them relatively at rest. Illustration : marine engines at sea.

Bodies have been spoken of several times as having the *same motion*. One body has the *same motion* as another if they could be rigidly connected without alteration of the motion. We have seen that this is a consequence of two bodies having no motion relatively to each other, or of their having the same motion relatively to a third. It is a well known theorem that a body may be moved from any position, to any other, by giving it two motions : one of translation through a certain distance, and one of rotation about a certain axis. Every motion, therefore, considered as a change of position, may be divided into two simple motions : one of translation, and one of rotation. In each of these cases the meaning of *same motions* can be easily understood. If a body have a motion of translation, another body has the same motion if it be translated parallel to the first, in the same *sense* and through an equal distance. Similarly if a body have a motion of rotation, another body will have the *same motion* if it be rotated through an equal angle in the same *sense* and about the same axis. As every motion can be resolved into translation and rotation, we may say th those motions are the *same* that are composed of *same* t lations and same rotations.

To illustrate: Suppose that $A\,B$ and $M\,N$ (Fig 4) are two bodies. Suppose $A\,B$ to move to $A_1\,B_1$. It is required to give $M\,N$ the same motion or change of position. $A\,B$ can be moved to $A_1\,B_1$ by translation to $A_1\,B^1$, and rotation about A_1 to the position $A_1\,B_1$. To give $M\,N$ the same change of position, it is only necessary to give it the same translation and the same rotation. Draw $N\,N_1$ and $M\,M_1$ parallel and $= A\,A_1$ and $B\,B_1$; $M_1\,N_1$ is position of $M\,N$ after the required translation. Draw $A_1\,M_1$ and $A_1\,M_2$ enclosing an angle $= B^1\,A_1\,B_1$ also draw $N_1\,A_1$ and $N_2\,A_1$, make the angle $N_1\,A_1\,N_2 = B^1\,A_1\,B_1$, make $A_1\,M_2 = A_1\,M_1$, and also make $A_1\,N_2 = A_1\,N_1$.

$N_2\,M_2$ will then be the position of $M\,N$ after the required rotation, and its change of position or motion has been the *same* as that of $A\,B$ moving to $A_1\,B_1$.

It is necessary to explain what is meant by *sense* of motion. A straight line may indicate the *direction* of motion of a point, *i. e.*, the point moves along the line; but it may move toward either end of the line, and the *direction* of motion be the same. If two points move along the line toward the same end of the line they move in the same *sense*.

In the case of rotation, a circle is the path of a point, but it may move "clockwise" or "anticlockwise." If two points move on a circle clockwise, they move in the same *sense*.

It has been seen that motion that is common to two bodies does not affect their relative motions. In studying relative motions of two bodies ∴ we may subtract common motions, *i. e.*, they need not be considered. But we may not only subtract motions but we may also add motions, and it is often very convenient. This is chiefly applicable to cases where two bodies are moving relatively to a third. Such problems may be at once simplified by adding to both bodies a motion equal but opposite to the motion of one body relatively to the standard.

Let it be required to find the relative motions of $A\,B$ and $M\,N$ during the motion $A\,B \ldots A^1\,B^1$ and $M\,N \ldots M^1\,N^1$ (Fig. 5). Motions that are common to two bodies do not alter their relative positions. If ∴ we give to both bodies a motion $A\,B \ldots A^1\,B^1$ in the opposite sense we shall n[illegible] change them relatively to each other. By doing this

have returned $A\ B$ to its original position; and have moved MN to $M_2\ N_2$. The relation of $A^1\ B^1$ to $M^1\ N^1$ is the same as of $A\ B$ to $M_2\ N_2$. But $A\ B$ is in its original position; therefore it is the same as if it had not moved at all. But MN is now found at $M_2\ N_2$. ... MN has had a motion MN ...$M_2\ N_2$ while $A\ B$ remained at rest; or the motion of MN relatively to $A\ B$ is MN... $M_2\ N_2$. We may apply this to a case that involves rotation and translation.

Let it be required to find the relative motion of AB and MN during the motion $A\ B$... $A^1\ B^1$ and MN... $M^1\ N^1$, (Fig. 6.)

Move A^1B^1 by translation to A_2B and give to M^1N^1 an equal translation to M_3N_3. Rotate A_2B about B to AB: give to M_3N_3 an equal rotation to M_2N_2. We have given to both the same motion, and ∴ have not changed their relative position. But AB is in its original position ∴ it is the same as if it had not moved ∴ all the motion of MN is motion relatively to AB, or the motion MN relatively to AB is MN ... M_2N_2.

This might just as well be done the other way, *i. e.*, we might as well determine the motion of AB relatively to MN.

In both cases the relative motions remained unchanged; because we gave to both the *same* motion and so gave them no relative motion. The motion of MN relatively to AB when AB is at rest is the same as the motion of AB relatively to MN when MN is at rest.

We may say then that A and B are two bodies that move relatively to each other, the motion of A relatively to B, is the same as of B relatively to A, and is the same whether both bodies be moving, or either one be stationary relatively to any particular standard. This sameness of motion, however, does not include *sense* of motion.

Motion has been spoken of as change of position, and it has been seen that it is necessary to consider change of position of one body relatively to another, and not absolutely changes of position. Before considering relative motion more at length, it is necessary to examine the general conditions by which relative position may be determined.

Just as the absolute motion of bodies in space does not concern us, so also the absolute position of a body in space, or a figure in a plane is something which requires no atten

A point or a figure may be assumed in any part of a plane, and the position of others relatively to it alone as of importance.

Starting then with the idea of a fixed point in a plane, (relatively fixed, not absolutely,) it may be stated that *the position of a point relatively to another is determined solely by the distance between them.* As long as the distance between the points does not undergo change, it is the same as if they were rigidly attached to each other, and the position of a point relatively to another to which it is rigidly attached cannot undergo change, no matter how much the line joining them may change its position relatively to the plane of motion.

Thus in figure 7 the position of A relatively to C is the same as of A^1 relatively to C, and in general all points in the circumference about C have the same position relatively to C. But clearly A and A^1 have different positions relatively to the plane. And so it may be said that *the position of a point relatively to a plane is not determined by its position relatively to a point in that plane.*

A straight line is, of course, fully determined when two of its points are known. *Therefore the position of a line relatively to a point is known if the position of two of its points relatively to the fixed point be known.*

Thus in Fig. 8, if A and B be known relatively to C, the position of the line is determined relatively to C. If $AC = A^1C$ and $BC = B^1C$ then the position of A^1B^1 relatively to C is the same as of AB; but obviously AB and A^1B^1 have different positions relatively to the plane. Therefore *the position of line relatively to a plane is not determined by its position relatively to a point in that plane.*

The position of a point relatively to a line is known *if its distances from two points of the line be known.* Thus in Fig. 9, if the distance CA and CB be known, C relatively to line AB is determined, but not fully, for there are always two points that fulfill both conditions, thus C and C^1. Therefore, for all the positions that a point in a plane may take relatively to a line, there are two positions. *Thus we see that the position of a point relatively to a plane is not absolutely determined by its position relatively to a line in that plane.*

The position of a line relatively to another in the same plane is determined if two points of the former are known relatively to two points of the latter.

Thus, in Fig. 10, if AC and AD are known, A is known relatively to CD, and if BD and BC are known, B is known relatively to CD. Then if A and B are known relatively to CD, the line AB is known relatively to CD. But, as in the last case, AB may take two positions in the plane for every position relatively to CD. Therefore *the position of a line relatively to a plane is not absolutely determined by its position relatively to a line in that plane.*

The position of a plane figure in a plane is known if the position of two of its points be known, *i. e.*, if the position of a line in it, relatively to two points in the plane, be known. Thus, let α (Fig. 11) be a plane figure, and AB any line in it; if A and B are known relatively to C and D the figure is determined, for if AB remain stationary, the only motion possible for α is about AB as an axis; but if it move about AB as an axis it will move out of the plane.

The relative *position* of points, lines, and plane figures in in a plane are determined by the conditions just discussed.

The *motion* of points and lines in a plane is the occupying of a series of positions relatively to another point or line. Each of these positions is determined by the above conditions. ∴ The conditions that determine the *position* of a point or a line in a plane relatively to another, determine also the *motion* of that point or line relatively to another.

The conditions that govern relative motion, corresponding those that govern relative position, may ∴ be stated very simply.

One point can move relatively to another only along a line joining them. The position of A (Fig. 12) depends simply on the distance AC. A can only change its position, *i. e.*, move, by changing the distance from A to C, or it can move only along a line, AC joining them. Thus A, in moving from A to E, does not move relatively to C, but if it moves from A to B, then it moves relatively to C, a distance $BC-DC$ along the line joining them.

The motion of a line relatively to a point is determined by the motion of two points in it relatively to that point. Each of these can move relatively to the point only along a line joining them. We can see then, just as in the case of relative position, that the motion of the line relatively to the plane is not determined by its motion relatively to a point in that

plane. The line might revolve about the fixed point and no motion relatively to it and yet have continuous motion relatively to the plane.

The motion of a point relatively to a line is determined by its motion relatively to two points of that line.

The motion of a line relatively to a plane, or a line of that plane, is determined by the motion of two points of one relatively to two points of the other.

The motion of a plane figure relatively to a plane is determined by the motion of any two points of it, i. e., a line.

This last may be stated in another way. The figure being supposed rigid, no part can move relatively to any other part, all parts then have the same motion, *i. e.*, the motion of the two points in the figure.

Having given then the motion of two points in a plane figure we know the motion of all other points, because we know it is the same motion, same being used in the sense already explained.

It has already been seen that if a body has plane motion ; the whole motion of the body is known, if the motion of one section parallel to the plane of motion is known. It has just been seen that the motion of a plane section is known if the motion of two of its points be known.

Therefore the plane motion of a body is known if the motion of its points, *i. e.*, a line in a section parallel to the plane of motion be known.

Thus all that has preceded that applies to the determination of the motion of a line, apply equally to the determination of the motion of a body.

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Motion has been considered thus far as a sequence of changes of position of finite extent. Each change occupied a finite interval of time, and at the beginning and end the body occupied different positions. But instead of studying these completed changes of position, it is often neccessary to examine the motion that a body has at a given instant. This is called the *instantaneous motion* of the body.

Every point in a moving body describes some curve in the plane of motion. These curves may be called *point paths.* In order to know the motion of a body completely we must know the paths of all its points; or of enough of them so that the others may be determined. To know the instantaneous motion of a body it is simply necessary to know the *direction* of the point paths at the given instant. By the direction of the point path at any instant is meant the direction in which the point that describes the path is moving at the given instant. It is the direction of a line joining the point at the instant with the next point in the point path that is infinitely near to it. But a line that joins two points of a curve that are infinitely near each other is a tangent to the curve. The direction of the instantaneous motion of a point moving in a curve is therefore a tangent to the curve through the point. Thus if a point be moving in a curve AB (Fig. 13,) and at the given instant be at C; the direction of its instantaneous motion is along a tangent to the curve at C or CD. The instantaneous motion of a point is known therefore if its direction of motion, *i. e.*, the tangent to its path be known for the given instant.

The instantaneous motion of a line is known if the direction of motion of two of its points be known.

It has been seen that the motion of a plane figure in a plane, is determined by the motion of any two points in the plane figure. This applies as well to instantaneous motion as to motion that involves a finite change of position. The only difference is that the motion is considered infinitely small. *The instantaneous motion of a plane figure is determined by that of two of its points. Also the directions of motion of all points in a figure are fixed when those of two points are fixed.*

Since the instantaneous motion of a point is known if its

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direction is known, the instantaneous motion is independent of the curve which is the path of the points motion.

Thus in Fig. 14 if *AB* represent the direction of instantaneous motion of the point *C* it will be clear that any curve drawn tangent *B* to *AB* at *C* might be the point path.

In machines, in many cases of parts that have quite complex motions, we are concerned only with the instantaneous motions; *i. e.*, with the directions in which points are moving at some given instant, and the complex motion may be replaced by the simple one which is for the instant identical with it, and which admits of treatment in the most direct possible way. To illustrate: Let *L* (Fig. 15) be any plane figure, and *A* and *B* two points in that figure. Let *a* and *b* represent the direction of motion of *A* and *B* if the direction of motion of two points in a plane figure be known, the instantaneous motion of the figure is determined. Let *AD* and *BO* be lines drawn from the points *A* and *B* at right angles to the lines of direction. About all points in *AO* as centres circles might be drawn tangent to the lines of direction at the point *A*, and any one of these circles is a possible path for the point *A*. Also about all points of *OB* as centres we could draw circles tangent to the line of direction at *B* and each one of these circles is a possible path for *B*. *A*, then, may have rotation for the instant, about any point of *AO*, and *B* also may have instantaneous rotation about any point of *BO*. But *A* and *B* are points of a plane figure, and therefore can have no relative motion. They must therefore have the same motion, *i. e.*, they must rotate about the same point as a centre. The possible centres of rotation of *A* are in *AO* and of *B* in *BO*. The centre of the instantaneous rotation of *A* and *B* is therefore the intersection of *AO* and *BO* or *O*.

It has been shown that the motion of a plane figure is that of any line of that figure. In this case the line *AB* is simply revolving about *O* as a centre and therefore the whole of the plane figure is rotating about a centre *O* which is called its *instantaneous centre* or as Prof. Kennedy prefers to call it, *virtual centre*.

The only assumption made is that the figure has plane motion. The points *A* and *B* are arbitrarily chosen and

might just as well be any other points, and there are no conditions regarding the form of their paths. The following conclusion is therefore perfectly general :

Whatever the motion of a figure in a plane it is always possible to find a point in that plane such that rotation about it shall be the same as that motion for the instant.

The motion of every figure in a plane must be at every instant a rotation about some point in that plane. This point, the *virtual centre*, will be denoted in the following work by the letter O.

All points in the figure must have the same motion and therefore they are all turning about O as a centre. The direction of motion of every point is therefore known, because it is simply the tangent to a circle drawn through the point with O as a centre. The line joining the point to the point O, is called the *virtual radius* of the point.

Stating this concisely we may say that when a figure is moving in a plane, the normals to the lines of direction of all of its points, *i. e.*, the virtual radii of all its points pass through a common point which is the virtual centre for its motion. *The virtual centre is therefore the intersection of the virtual radii of all points in the figure.*

One special case may be noticed. Suppose that the lines of direction of all the points of a plane figure are parallel, the normals or virtual radii would then be parallel, and the virtual centre would be at an infinite distance.

The plane figure α, (Fig. 15,) is rotating about O. The figure may be of any form and may or may not include O. If it does include O the question comes, what is the motion of O. As a point in a rigid figure it has the same motion as all other points and therefore it turns about O; in other words it is a point turning about itself, that is it is not changing its position relatively to the plane. *The virtual centre of a figure relatively to a plane is always a fixed point of the figure.* Only one such a point can be fixed, for to fix two would be to fix a line; and to fix a line would be to fix the figure. But one such point is always fixed and that point is always the virtual centre.

O is the virtual centre of the figure relatively to the plane. But it is also the virtual center of the plane relatively to the

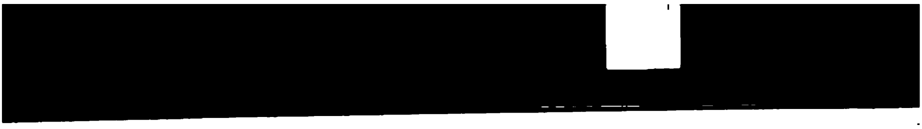

figure. We have seen that even where the motion of A and B is of quite a general kind, the motion of A relatively to B is the same as of B relatively to A. In this case the motion of α is simply a rotation about O relatively to B, therefore the motion of B is a rotation about the same point relatively to a.

The virtual center that characterizes the relative motion of two bodies is the point at which they might be connected for the instant. If our imaginary figures were replaced by actual bodies the virtual center is the point through which might run the axis of a pin or shaft connecting them. If only there were means of changing this connection from instant to instant.

This matter may be summed up as follows:

Whatever the real motions of two bodies may be, it may be fully represented at any instant, by a simple rotation about a determinate point, which may be called the virtual centre of the relative motions of the two bodies. This point is for the instant a point common to the two bodies; a point at which they may be supposed to be physically connected. It is a point in each body that has no motion relatively to the other.

In general we need to consider that a figure is rotating at different instants about different centres; but in one case this is not so; it is the case where a figure rotates continuously about the same centre. We have such a case in a pulley; here the virtual centre becomes a permanent centre. If we have to do with the figure in one position at a time and not through a series of positions, the case is not different from instantaneous centre; and every proof or construction that applies to one applies equally well to the other. *Permanent centre* may then be considered as a special case of *virtual centre.*

In general, however, a body moving in a plane does not continue to turn about its virtual centre. On the contrary if several positions of a figure be taken, as a, b and c (Fig. 16), and the virtual centres found for each position, we shall find them occupying as many positions as the figure.

If we suppose the figure to change its position continuously then the virtual centre will change its position also *continuously*, or will describe a curve which is the locus of *virtual centres* or the *centrode*, as it is called.

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If the figure be assumed to move relatively to the plane, the latter being fixed, the virtual centres will describe a centrode on the fixed plane. If we suppose the figure fixed and the plane moving with the same motion, the virtual centres of the plane relatively to the figure will describe some different centrode on the figure. But in either case the relative motion is the same ; and since the virtual centre is a point at which the two bodies may be physically connected for the instant. We see that the points of one centrode will touch the points of the other successively, as the motion goes on ; or, in other words, the centrodes will roll on each other as the bodies go through their succession of relative positions. Having given then the centrodes of the relative motion of two figures, by rolling these centrodes on each other, the entire relative motion of the two figures may be determined.

An example may serve to make this clear.

Suppose a circle to roll along a straight line. The instantaneous axis of the circle at any time is the point at which it touches the straight line, because, at that instant every point in the circle is rotating about that point. As the circle rolls along the line the locus of its virtual centres on the plane of the line is the line itself. Suppose now that the circle is fixed and that the line moves on it in such a way that the relative motion is the same as before, the instantaneous axis of the line relatively to the fixed plane of the circle is at the point where they touch. The locus of these virtual centres is, therefore, the circle. In the special case chosen the centrodes are the same as the bounding lines of the figures themselves. It is clear that we can roll these centrodes on each other, and by so doing show completely the relative motion of the two figures.

But we might have just as well taken two lines to represent the figures and we could have moved them so that, as before, the centrodes would be the circle and line, or some other curves, and by rolling them on each other we could have represented competely the relative motion of the bodies represented by the lines.

The nature of plane motion having been studied, the means by which this plane motion may be obtained in machines completely constrained next needs attention.

It has been seen that the motion a body has, if it be of

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suitable material, depends on the *form* of its connection with other bodies. This form has to be such as not only to *allow* the required motion, but also to render any other motion impossible. It may be proved that any plane motion whatever may be constrained; but we are concerned at first with the constraining of translation and rotation, and so need only to study the means by which this is accomplished.

Case I. Let α and β (Fig. 17), be two bodies, and let it be required to constrain the motion so that α shall be allowed to rotate relatively to β about C as a centre; and also so that no other relative motion of α and β shall be possible. Upon the back of α let a cylindrical projection be formed whose axis is at right angles to the plane of motion and passes through C. Let a circular hole be made in β of diameter = the cylindrical projection on α; and whose axis is also at right angles to the motion plane, and coincident with C. These cylindrical surfaces may now be placed in contact, and motion of rotation about C is possible. But if force be applied at right angles to the plain of motion, α and β may be separated, *i. e.*, may have relative motion, other than rotation about C.

To complete the constrainment, a piece A may be made fast to the cylindrical projection on α as shown.

Case II. In Fig. 18, let α be a body that is required to have rectilinear motion relatively to β, parallel to the edge AB. Upon the back of α a rectangular projection may be formed, and in β a slot may be cut whose width equals that of the projection on α. The projection may be inserted in the slot, and the required motion is possible. As in the other case however α and β may be separated at right angles to the motion plane, and this possible relative motion may be prevented just as before, and the constrainment rendered complete. The two forms supplied to α and β in each case to constrain their relative plane motion, are called a *pair of Kinematic Elements*. It will be seen that from their nature, they must always exist in pairs; one element can no more constrain motion, than one body can constitute a machine. The lowest terms to which a machine may be reduced is a pair of Kinematic Elements.

The pair used in Case I is called a *turning pair*; that used

in Case II is called a *sliding pair*. The turning pair is for the constrainment of rotary motion. It takes the form of a solid of revolution, its cross section being such that axial motion is impossible. Its commonest form is the "pin and eye," or the "journal and bearing."

The sliding pair is for the constrainment of motion of translation. Its form is essentially prismatic, *i. e.*, it is a solid having plane sides parallel to the direction of required motion.

These two particular pairs are specially valuable to the engineer for two reasons: 1st, because they have forms that are easily produced in the shops. The lathe is the most common of all the machine tools, and the production of cylindrical surfaces in the lathe the simplest of the shop processes, and these are the forms necessary for the turning pair. Next in facility and economy of production to cylindrical surfaces, are the plane surfaces produced by the planer or shaper, or milling machine; and these are the surfaces necessary for the sliding pair. The second reason that turning and sliding pairs are valuable is, that the contact of the two elements with each other is *surface contact*. In many cases the contact between elements is only along a line, or a few lines of the elements, as in cams or toothed wheels. Geometrically the constrainment is just as complete as in the turning or sliding pairs. But we must bear in mind that in machines the parts are subjected to wear, and that that constrainment is most perfect which is least apt to be deranged by the wearing away of the surfaces of the constraining pairs, and this condition occurs where the pressure is distributed over the greatest extent of surface, or where we have surface contact instead of line contact. Pairs in which there is surface contact are called *lower pairs*. Those in which there is line contact are called *higher pairs*. The surface contact spoken of is possible in only three classes of surfaces, 1st, surfaces of revolution; 2d, plane surfaces; 3d, cylindric screw surfaces. The first is utilized in the turning, the second in the sliding, and the third in the twisting pair. This latter motion is not plane motion and so does not need to be considered here. The only plane motions, therefore, that can be constrained by pairs having surface contact ar

rotation and rectilinear translation. For all other plane motions we must resort either to higher pairs with line contact, or to indirect constrainment with lower pairs.

This brings us to the consideration of *links*, *chains*, and *mechanisms*.

In order that the relative motion of two bodies may be constrained, they must be connected by a suitably formed pair of elements. This combination is the simplest form that can be treated as a machine, and yet it may be a machine.

In the case of two bodies connected by a pair of elements, each carries but one element. With this limitation nothing further can be obtained. In order that more than two bodies may be combined into a machine, it is necessary that each should carry at least two elements, and this number can be increased indefinitely.

Let bodies carrying but two elements be considered first.

It is an essential condition of a machine, and, therefore of the combination of bodies with which we have now to deal, that at no instant shall it be possible for any one of the bodies that form it, to move in more than a single way. If any other motion were possible for it at that instant, the motion that would result would be dependent on the direction and magnitude of the forces causing the motion. It would not then be constrained motion.

If at any instant the bodies a, b, and c, of a machine are movable, and if a and b can move or not move while c is moving, and it is only a question of the force acting whether c moves alone or takes a and b along with it. But the relative position of a and b to c in the two cases would be entirely different and the position of c at any instant relatively to a and b would be dependent upon the resultant force acting upon c, and therefore could be altered at any instant by a change in this force, and c would not be constrained relatively to a and b and the combination could not form any part of a machine.

To illustrate. Fig. 19 shows 5 links connected by turning pairs. If a force P be applied as shown, motion will result. If while P is acting d be held, a will turn about O. If a be held, d will turn about O'. The motion then depends on the forces applied and therefore the constrainment is not com-

plete; and the combination will not serve the purpose of a machine from definition.

The obvious statement may be made, that every body that forms a part of a machine, must be constrained relatively to all other bodies in the machine.

The question arises, can a machine contain bodies of single as well as double elements. It cannot, because the body of the single element is paired with only one body, and its motion is constrained only relatively to that body, and not to the other to which it may be indirectly connected. Thus, suppose the links, *a*, *b*, and *c* (Fig. 20), are connected by turning pairs at *O* and *O'*. *a* is constrained relative to *b*, and *c* is also constrained relative to *b*; but *a* is not constrained relatively to *c*, nor *c* relative to *a*. The conditions of the definition of a machine are not met therefore, and this cannot be a machine.

For the present then only bodies of two elements are to be dealt with in the building up of machines.

Bodies that are arranged, by being provided with two or more suitably formed elements, to form part of a machine are called *Kinematic links*, or *links* simply.

In the building up of a machine of these links it is necessary (calling them *a*, *b*, *c* and *d*) that one element of *a* should pair with one of *b*, the second of *b* with the first of *c*, and so on. On the last element there will be left one element unpaired, and the first link also has an unpaired element. The arrangement is completed by pairing these with each other, and the result is called a *closed kinematic chain* or simply a *chain*.

In this each link is paired with two others, its adjacent links. Its motion relatively to each of those is completely constrained, therefore, and also it is constrained relatively to the non-adjacent links, because obviously it can have but one motion relatively to them.

To sum up: The simplest combination having the nature of a machine is obtained by connecting two bodies of suitable material by means of such geometric forms as to completely constrain their motion. These constraining forms are called ***kinematic elements***, and must exist always in *pairs*. If
contact be along a line or a few lines, these pairs are

higher pairs; if the contact be throughout the entire surface of the elements, they are called *lower pairs.* Two lower pairs only are available for the constrainment of plane motion, the *turning pair* and the *sliding pair.*

When a constrained combination is made up of more than two bodies, each of them must carry two elements, at least, belonging to different pairs. Such bodies are called *links.*

A series of such links so connected that each element in each one is paired with its partner in another, and so on till the motion of every link is completely constrained relatively to every other, is called a *kinematic chain.* If we *fix* one of the links of a chain, *i. e.*, make it the standard to which the motion of the others is referred, it becomes a *mechanism.* A mechanism is the ideal form of a machine, and represents it fully for kinematic problems.

In converting a Kinematic chain into a mechanism any one of the links may be fixed, and hence as many mechanisms may be derived from a chain as it has links. In the *slider crank chain* all are different; this is not always the case though, two or more may be similar or identical.

Chains built up of links having each two elements are called *simple chains. Compound chains* are such as have some links that have more than two elements. All that has been said about simple chains applies as well to compound chains with slight verbal alteration.

Fig. 21 shows a compound chain, the links a and d each carrying three elements.

To determine the number of virtual centres in any kinematic chain, the number of links being known, $=n$. Every link must have a virtual centre relatively to every other and therefore there are $n(n-1)$ virtual centres. But the virtual centre of a relative to b is the same as of b relative to a, *i. e.*, O_{ab} and O_{ba} are the same point, or in other words each virtual centre is double, and therefore the real number of points that are virtual centres is $\frac{n(n-1)}{2}$

The principles of virtual centers may now be applied to mechanisms.

Fig. 22 shows simplest case; viz., two links joined by a *turning pair.* The motion of a relatively to b is a turn

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O. Every point of a therefore turns about O, and the centre of its motion at any instant is O, therefore the instantaneous or virtual centre of a relatively to b is O, and we may call it O_{ab}. By the same reasoning if we fix a we have the virtual centre of b relatively to a at O, or it is O_{ba}.

Let the lever crank chain (Fig. 23), be next considered.

It is required to find the instantaneous centre of each link relatively to every other. The number of virtual centres is 6. Obviously from what has preceded, the virtual centres of adjacent links, O_{ab}, O_{bc}, O_{cd} and O_{ad}, can be at once located. But there are two others to be determined, viz., O_{ac} and O_{bd} or those of opposite links. Consider O_{bd} first. The motion of every point of a relatively to d is known; but a and b have a common point and this point can have but one motion relatively to d. Therefore its motion as a point in b relatively to d is known. Also the motion of every point in c relatively to d is known, but c also has one point in common with b. Therefore the instantaneous motion of two points in b relaatively to d is known. The points are O_{ab} and O_{bc}. Obviously the normal to the direction of motion of O_{ab} is the line of the link a; therefore O_{bd} is somewhere in this line. Also the normal to the direction of motion of O_{bc} is the line of the link C; therefore O_{bd} is somewhere in this line. Since it is in the line of a and c it must be at their intersection. O_{bd} is therefore the virtual centre of b relatively to d and of d relatively to b. Exactly similar reasoning would locate O_{ac}.

The general statement may now be made that in a chain consisting of four links, connected by four turning pairs, the virtual centre of either pair of opposite links is at the intersection of the axes of the other pair, and the virtual centre of any pair of adjacent links is the intersection of their own axes or, it is their permanent centre.

Inspecting this chain a peculiarity appears. Taking any three links of the chain, we see that the three virtual centres corresponding to the three links lie in a straight line. This is not a mere coincidence, but results from a general law, as may be proved very simply.

Let a, b, and c (Fig. 24), be any bodies having plane motion and let O_{ab}, O_{bc}, and O_{ac}, be the virtual centres for their motion.

O_{ac} is a point common to a and c. As a point in a it is moving about O_{ab} relatively to b and as a point in C it is moving about O_{bc}. As a point in a its direction of motion is at right angles to a line joining it to O_{ab}. As a point of C its direction of motion is at right angles to a line joining it to O_{bc}. This point can have but one direction of motion and therefore the lines joining it to O_{ab} and O_{bc} are at right angles to the same line, and hence must be either parallel or coincident. But they both pass through the same point O_{ac}, therefore the lines must be coincident and the three points O_{ab}, O_{bc}, O_{ac}, are in the same straight line. We may thus formulate this law. If any three bodies have plane motion their virtual centres all lie in one straight line.

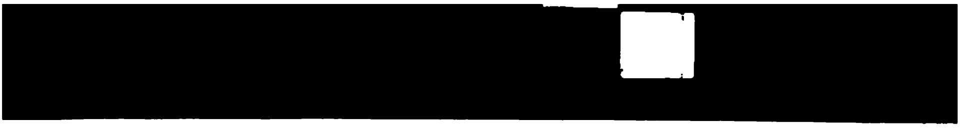
The virtual centres of the slider crank chain may now be determined. (Fig. 25.) By simple inspection the virtual centres of the adjacent links O_{ab}, O_{bc}, O_{cd}, and O_{ad} may be located.

In this case the motion of C relatively to d is a motion of translation or of rotation about an infinitely distant centre.

It is required to determine the virtual centres of opposite links b, d, and a, c. As before the motion of a and c relatively to d is known, and the link b has a point in common with each of these ; the motion of two points of b relatively to d is therefore known and the virtual center of b relatively to d will be at the intersection of the virtual radii of these points. The line of the virtual radius of the point O_{ab} is the line of the axis of a. The virtual radius of O_{bc} is a line joining that point to O_{cd} at an infinite distance and is therefore a line at right angles to d; the intersection of these radii O_{bd} is therefore the virtual centre of b relative to d.

To find O_{ac}, the motion of every point of b relatively to c is known, and the normal to the direction of motion is the axis of the link b. The motion of d relatively to c is also known, and a has one point in common with d and the normal to the direction of motion of that point is a line at right angles to d, the intersection then of these two normals is O_{ac}. This may be proved somewhat more simply by the help of the theorem just demonstrated. For from the theorem since a, b, and c are three links having plane motion, and since O_{bc} and O_{ab} are two of their virtual centres, the other one O_{ac} must be in the same straight line. Also since a, d and c are three links having plane motion, and since O_{ad} and O_{cd} are two of their virtual centres the other, O_{ac} must be in the same straight line. Clearly O_{ac} is at the intersection of these two straight lines.

The use of this theorem, therefore, greatly shortens the proof in the case of simple mechanisms; but the proof can usually be made without it. But in case of links in compound chains it is almost indispensable.



Suppose that we have a four link mechanism, the links being *a*, *b*, *c* and *d*. Let *d* be fixed.

Let *a* be joined to *d* by a turning pair.

Let *b* be joined to *a* by a turning pair.

Let *c* be joined to *b* by a sliding pair.

Let *d* be joined to *c* by a sliding pair.

This mechanism is known as the "slotted cross head," and is shown in Fig. 26.

Let the six virtual centres of this mechanism be found. The four virtual centres O_{ad}, O_{ab}, O_{cd} and O_{bc} may be located at once. O_{ac} and O_{bd} remain to be found. *a*, *b* and *c* are three bodies having plane motion, therefore, O_{ab}, O_{bc}, O_{ac} are in the same straight line. Considering *a*, *c* and *d*, O_{ad}, O_{cd}, O_{ac} are in the same straight line. O_{ac} is, therefore, at the intersection of O_{ab} O_{bc} and O_{ad} O_{cd}.

To find O_{bd} consider first *a*, *b* and *d* then,

O_{ab}, O_{ad}, O_{bd} are in the same straight line; also considering *b*, *c* and *d*, O_{bc}, O_{cd}, O_{bd} are in the same straight line.

But the line joining O_{bc} to O_{cd} is wholly at infinity.

Therefore O_{bd} is on the line O_{ad} O_{ab} at infinity.

In the case of four links connected by turning pairs into a closed chain, it has been seen that the virtual centre of adjacent links is at the centre of the pair that joins them; or at the intersection of their axes. Of opposite links, the virtual centre is at the intersection of the axes of the other two.

This, of course, applies equally well in all positions of the chain. To illustrate, let *a* be moved down so that *b* crosses *d*. (Fig. 27.) O_{ac} and O_{bd} will be located as shown.

When the link *a* coincides with *d* (Fig. 28), O_{ac} coincides with O_{ab}, and O_{bd} coincides with O_{cd}.

In the slider crank chain, when *a* coincides with *d* the chain is represented by a straight line, (Fig. 29). O_{ac} then coincides with O_{ad}, and O_{bd} with O_{bc}.

It is required to find all the virtual centres in the compound chain, Fig. 30. The part above the line *EF* is a slider crank chain, and its motion is in no way affected by the attachment of the part below *EF*; on the other hand, the lower part is a lever crank chain, and its motions are not affected by the upper part. The chain may be treated in two parts, and all the virtual centres of each part may be located at once, from the preceeding considerations. Each pa

have six virtual centres and twelve would be obtained in this way ; but that O_{ad} is common to both parts, and therefore but eleven are found. The entire chuin has six links and therefore the number of virtual centres

$$\text{Virtual centres} = \frac{n(n-1)}{2} = \frac{6 \times 5}{2} = 15.$$

Therefore there remain four to be determined. They are O_{be}, O_{ef}, O_{bf} and O_{ce}. Let O_{be} be found first. *a*, *b* and *e* are three bodies having plane motion and therefore, O_{ab}, O_{be} and O_{ae} are in the same straight line, therefore O_{be} is in the line *AB*. Next considering *b*, *d*, and *e*, it follows that O_{bd}, O_{de} and O_{be} are in the same line and that line is *CD*. Therefore O_{be} is at the intersection of *AB* and *CD*. The remaining three virtual centres may be found in precisely the same manner.

The determination of the direction of motion of any point, at any instant, in a kinematic chain is now easy.

Every point in a link, is moving relatively to any other link, along a line at right angle to the line joing the point, to the virtual centre of the relative motion, of the two links concerned.

Let us consider a simple chain, assume points in one of the links, and find the direction of motion of these points relatively to all the other links. (Fig. 31.)

Let the link *d* be enlarged into a plane figure and take the points x x' and x'', first consider x. Its direction of motion relatively to *b* is at right angle to a line joining it to O_{bd} or d_b. Similarly its motion relative to *a* is d_a and relative to d_e The same for the other points.

The relation of motion to the unit time has been thus far disregarded. Motion per unit time is velocity.

Many kinematic problems can be treated independent of velocity ; but there are others that require its consideration.

The *absolute* velocity of any point in a machine depends upon the forces acting, as do also the *changes* of velocity. With these we have nothing to do as yet. But the *relative* velocities of different points in a machine, at any given instant, can be determined by purely geometrical considerations, and so the treatment comes under the head of Kinematics.

Consideration of the subject of virtual centres, has

us that every body in a machine, has a motion that is equal for the instant to a rotation about a given point. It has also been seen how this point may be found. A machine part is then in exactly the condition for the instant of a wheel turning about its centre. Obviously then the velocity of any point is proportional to its distance from the centre of rotation; or, in other words, to its virtual radius.

If then the velocity of any point be given, that of others is determined by finding the virtual centre, and the distance of the various points from it. Or even if no velocities be given, the relative velocity of any two points may be determined.

This of course is in the case of linear velocity. It is also necessary to consider angular velocity. When a body is rotating about any fixed axis its motion is characterized by two conditions.

1st. All points in the body have the same angular velocity.

2d. The linear velocity of all points is proportional to their distance from the fixed axis. All points at the same distance from the axis, have the same linear velocity.

But these conditions are true of rotation independent of its duration, and therefore apply as well to instantaneous rotation, as to continuous rotation. Obviously then every point in a link at a given instant has the same angular velocity; every point *i. e.* sweeps through the same angular distance in the same time. And the linear velocities of different points are proportional to the virtual radii of the points.

To illustrate: In Fig. 25, every point in the link a is turning with the same angular velocity about O_{ad}. Also every point in b is turning about O_{bd} with the same angular velocity; also any point in a at a given distance from O_{ad}, moves with half the linear velosity of a point twice as far from O_{ad}, and with twice the linear velocity of a point half the distance from O_{ad}. This is true of any link whether the turning be about a permanent, or a virtual centre, and so is true of b.

This makes it easy to find the velocity of any point in a link, if that of any other be known.

In a "slider crank chain" for instance, Fig. 32, if the velocity of the *point A*, be given, then the velocity of A' is *to that of A* as the virtual radii are to each other.

$$\text{Or, } \frac{\text{vel. of A}}{\text{vel. of A}'} = \frac{\text{OA}}{\text{OA}'} = \frac{\text{virtual radius of A}}{\text{virtual radius of A}'}$$

Or this may be done graphically.

From A in any convenient direction draw AB representing volocity of A on some scale. Through B and the virtual centre O_{ad} draw a line. Through A' draw a line parallel to AB and cutting $O_{ad}B$ in B'.

$O_{ad}AB$ and $O_{ad}A'B'$ are similar triangles whose corresponding sides are virtual radii of A and A'- Their bases are then proportional to these virtual radii. Therefore,

$$\frac{\text{veloc. A}}{\text{veloc. A}'} = \frac{\text{AB}}{\text{A}'\text{B}'}$$

Or take this case Fig. 33. Let the velocity of A in the link b, d being fixed be given, and the velocity of A' required. Join A and A' to the virtual centre of b relatively to d. Then

$$\frac{\text{veloc. A}}{\text{veloc. A}'} = \frac{\text{O}_{bd}\text{ A}}{\text{O}_{bd}\text{ A}'}$$

Or, suppose the velocity of $A{=}AB$, join $A\ A'$ and draw BB' parallel to AA'; from similar triangles we have

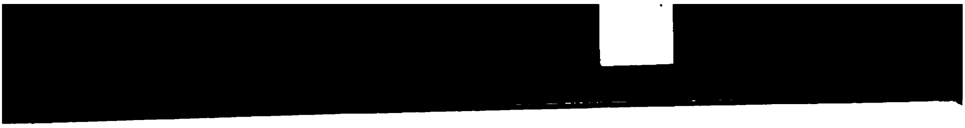
$$\frac{\text{virt. rad. A}}{\text{virt. rad. A}'} = \frac{\text{O}_{bd}\text{ A}}{\text{O}_{bd}\text{ A}'} = \frac{\text{BA}}{\text{B}'\text{A}'} = \frac{\text{veloc. A}}{\text{veloc. A}'}$$

This same method is just as applicable to points in different links.

To make this application it is necessary to use the theorem, that the virtual centre of any link, relatively to any other, is a point common to both.

Suppose that the velocity of A is given, Fig. 34, and that of B is required. If we find what the velocity of O_{ab} is as a point in a, we of course shall know what it is as a point in b. Let AA' be the given velocity of A, join A' and O_{ad}, the virtual centre of a relatively to d; through O_{ab} draw a line parallel to $A'A$ or $O_{ab}C$; this will represent the velocity of the point O_{ab} either as a point of a or b. But the virtual centre of b is O_{bd}, join C and O_{bd}; transfer B' to B." We now have similar triangles whose corresponding sides are the virtual *radii, of the points* O_{ab} and B.

The bases of the triangles are proportional therefore to the



virtual radii and the base CO_{ab} = velocity of point O_{ab}, therefore $B''B'$ represents velocity of B.

This might be obtained without construction.

$$\frac{\text{veloc. of } O_{ab}}{\text{veloc. of A}} = \frac{O_{ad}\,O_{ab}}{O_{ad}\,A}$$

$$\therefore \text{veloc. } O_{ab} = \text{veloc. A} \times \frac{O_{ad}\,O_{ab}}{O_{ad}\,A}.$$

Thus we have the velocity of a point in b, and we may get the velocity of any point in b by multiplying by the ratio of the virtual radii of the points. Thus the velocity of

$$B = \text{veloc. } O_{ab} \times \frac{\text{rad. B}}{\text{rad. } O_{ab}} = \text{veloc. A} \times \frac{O_{ad}\,O_{ab}}{O_{ad}\,A} \times \frac{\text{rad. B}}{\text{rad. } O_{ab}}$$

This applies just as well to opposite as to adjacent links.

Suppose in Fig. 35 that the velocity of A is given, and of C required. O_{ac} is a point common to A and C, and therefore its velocity must first be found. Transfer, for convenience, A to A' making $O_{ad}A' = O_{ad}A$. The velocity of O_{ac} is to that of A' as their virtual radii; or,

$$\frac{\text{veloc. } O_{ac}}{\text{veloc. } A'} = \frac{O_{ad}\,O_{ac}}{O_{ad}\,A'} \;\therefore\; \text{veloc. } O_{ac} = \text{veloc. } A' \times \frac{O_{ad}\,O_{ac}}{O_{ad}\,A'}$$

but O_{ac} has the same velocity as a point in C, therefore the velocity of C ts to that of O_{ac} as their virtual radii relatively to d; or,

$$\frac{\text{veloc. } C}{\text{veloc. } O_{ac}} = \frac{O_{cd}\,C}{O_{cd}\,O_{ac}}$$

Therefore,

$$\text{veloc. } C = \text{veloc. } O_{ac} \times \frac{O_{cd}\,C}{O_{cd}\,O_{ac}} = \text{veloc.} A' \times \frac{O_{ad}\,O_{ac}}{O_{ad}\,A'} \times \frac{O_{cd}\,C}{O_{cd}\,O_{ac}}$$

This may be solved graphically. Lay off from A' a line $A'A''$ representing the velocity of A. Draw a line through O_{ad} and A'', then $O_{ac}D$ will represent the velocity of O_{ac}. Considering O_{ac} a point in the link C, its virtual radius $= O_{cd}$ O_{ac}, and the virtual radius of C is CO_{cd}. Transfer C to C' making $O_{cd}C' = O_{cd}C$. Then if $O_{ac}D$ represent the velocity of O_{ac} as above, $C'C''$ drawn parallel to O_{ac}D, will represent the velocity of C' and therefore of C.

In the cases *thus* far considered, the virtual centres necessary for the determination, have fallen within the limits of the

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paper; but in many cases one or more of these virtual centres may be inaccessible, and a modification of the methods is necessary.

Consider first Fig. 36. The velocity of the point O_{eb} is given, and that of O_{ed} is required. O_{ac} is inaccessible. Through any point of the link b, as A, draw a line AB, parallel to the link d. The triangle $AB\,O_{cb}$, is similar to the triangle $O_{cb}\,O_{cd}\,O_{ac}$ whose sides cannot be measured. Therefore AB is proportional to the virtual radius of the point O_{cd}, and AO_{cb} is proportional to the virtual radius of the point O_{cb}. The ratio of these sides is therefore the ratio of these virtual radii equal to the ratio of the velocities of O_{cb} and O_{cd}. Therefore

$$\frac{\text{veloc. of } O_{cb}}{\text{veloc. of } O_{cd}}=\frac{A\,O_{cb}}{AB}.$$

If the velocity of O_{cb} be given $=$ to a line GH, then to determine the velocity of O_{cd} on the same scale, lay off $AD = GH$, and draw DD' parallel to $O_{cb}\,O_{cd}$. Then AD' is the required velocity of O_{cd}.

In this case the points selected are such, that the direction of their virtual radii is known. Fig. 37. Suppose that the relative velocity of A and B is required. Draw EF parallel to CD, divide the line EF into parts that are proportional to the parts into which the line CD is divided by the points A and B. Let G and H be the points of division. From geometry it follows, that a line drawn through A and G, would pass through O_{ac}, which is the intersection of the links b and d. Also similarly, the line BH passes through O_{ac}. The virtual radii of A and B are therefore AO_{ac} and BO_{bc}, whose direction is determined. But, since CD and EF are parallel,

$$\frac{AG}{BH}=\frac{AO_{ac}}{BO_{ac}} \text{ and } \therefore = \text{required velocity ratio} = \frac{\text{vel. } A}{\text{vel. } B}.$$

In the chain, Fig. 38, O_{ac} is inaccessible, and the relative velocity of points in the links a and c, is required.

Join O_{cd} to O_{ab} and draw MN parallel to d in any convenient position. The lines AB and BC have the same ratio to each other as $O_{ad}\,O_{cd}$ and $O_{cd}\,O_{ac}$. It is this ratio that is needed in the solutions, and we may proceed just as in the similar problem already solved.

Let these results now be put into a more general form. The general problem is, *given the linear velocity* v_1 *of a point* A *in a link* a, *to find the linear velocity* v_2 *of a point* C, *in another link* c, *of the same mechanism, the fixed link being* d.

Let us look again at two cases. (See Fig. 39.)

Here $v = \frac{v_1\, O_{ac}\, O_{ad}}{A\, O_{ad}}$ also $v_2 = \frac{v\, C\, O_{cd}}{O_{ac}\, O_{cd}}$

$$\therefore\ v_2 = \frac{v_1\, O_{ac}\, O_{ad}}{A\, O_{ad}} \times \frac{C\, O_{cd}}{O_{ac}\, O_{cd}}$$

$$\therefore\ \frac{v_2}{v_1} = \frac{O_{ac}\, O_{ad}}{O_{ac}\, O_{cd}} \times \frac{C\, O_{cd}}{A\, O_{ad}}.$$

In Fig. 40 we have,

$$v = \frac{v_1\, O_{ac}\, O_{ad}}{A\, O_{ad}}$$

$$v_2 = \frac{v\, C\, O_{cd}}{O_{ac}\, O_{cd}}$$

$$\therefore\ v_2 = \frac{v_1\, O_{ac}\, O_{ad}}{A\, O_{ad}} \cdot \frac{C\, O_{cd}}{O_{ac}\, O_{cd}}$$

$$\frac{v_2}{v_1} = \frac{O_{ac}\, O_{ad}}{O_{ac}\, O_{cd}} \cdot \frac{C\, O_{cd}}{A\, O_{ad}}.$$

A similarity is seen in these results, and it may be said,

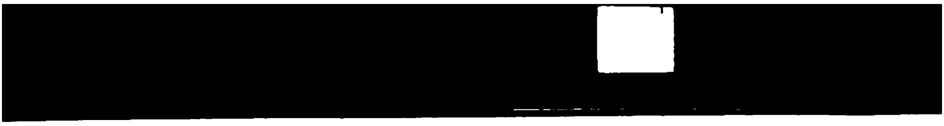
$$\frac{\text{vel. } C}{\text{vel. } A} = \frac{v_2}{v_1} = \frac{O_{ac}\, O_{ad}}{O_{ac}\, O_{cd}} \cdot \frac{C\, O_{cd}}{A\, O_{ad}}.$$

This in words is : *The velocity of* C, *is to the velocity of* A, *directly as the virtual radii of those two points relatively to the fixed links, and inversely as the virtual radius in* c *and* a *of the common point of the two links.*

If the two points are in one link the first term of the second member of the equation goes out, and the velocities of the points are to each other as their virtual radii relatively to fixed links.

To make clearer the determination of relative linear velocity, in kinematic chains, let two problems be solved in the slider crank chain.

Suppose the linear velocity of A in a (Fig. 41), is given, and the linear velocity of C in the link c is required. The link c relatively *to* d, *which* is the fixed link, rotates about a point *at an infinite* distance, and therefore all points in c have



same velocity, and therefore any point of c may be selected, and the result will be the same. Let us choose the centre of the turning pair that connects b to c. This is a point that remains common to b and c and therefore as a point in b it has the same linear velocity as all points of c and so our problem becomes, the velocity of a point A given, to determine the velocity of a point B in b adjacent. a and b have O_{ab} common, therefore velocity O_{ab} = velocity

$$A \cdot \frac{O_{ab}\, O_{ad}}{O_{ab}\, A}.$$

The velocity of

$$B = \text{veloc. } O_{ab} \cdot \frac{O_{bd}\, B}{O_{ab}\, O_{bd}} = \frac{O_{bd}\, B}{O_{ab}\, A} \cdot \frac{O_{ab}\, O_{ad}}{O_{ab}\, O_{bd}} \cdot \text{veloc. } A,$$

$$\therefore \frac{\text{veloc. } B}{\text{veloc. } A} = \frac{O_{bd}\, B}{O_{ab}\, A} \cdot \frac{O_{ab}\, O_{ad}}{O_{ab}\, O_{bd}}.$$

This formula corresponds to the general form just derived.

Suppose that the given velocity is that of the point B in the link b. (Fig. 42) and the required velocity is that of the point C, which may be any point in link c, since all points in c have the same velocity relatively to d. As before the centre point B' of the turning pair that connects b and c may be seected, and this is a point in b. The problem then becomes; given the linear velocity of one point in a link; required linear velocity of another point in same link. Of course the velocities are proportional to the radii, or distance from the points to virtual centre of the motion of the links relatively to the fixed link.

$$\text{Thus, } \frac{\text{veloc. } B}{\text{veloc. } B'} = \frac{B\, O_{bd}}{B'\, O_{bd}}.$$

ANGULAR VELOCITY.

Linear velocity is expressed in different units as feet, meters or miles per unit time. Angular velocity also may be expressed in different units. In engineering the unit is *one revolution per minute.* Thus a shaft is said to have an angular velocity of 30 when it makes 30 revolutions per minute. To find the linear velocity of a point, it is necessary simply to multiply the angular velocity in this system of units by $2\pi r$, *i. e. by the length* of the circumference of a circle *in which the point is moving.*

For mathematical purposes the unit of angular velocity is *the movement through an arc whose length = its own radius, per unit time.*

To convert angular velocity in this second system of units into linear velocity it is necessary to multiply by r.

To convert velocities given in the first system of units into those of the second.

In first, angular velocity $\times$ $2\pi r$ = linear velocity.

In second angular velocity $\times$ r = linear velocity.

Therefore, if angular velocity in the first system, be divided by 2π, it will equal angular velocity in the second.

If two points in different bodies, have same radius and equal linear velocities, their angular velocities are equal; and if they have equal radii, and unequal linear velocities, the angular velocities will be proportional to the linear velocities.

Also if two points have the same linear velocities, and different radii, the angular velocities will be inversely proportional to their radii.

In general therefore, the angular velocities of two points in different bodies, are proportional directly to their linear velocities, and inversely to their radii. But since all points in a body must have at each instant the same angular velocity we may say that the angular velocities of any two bodies having plane motion, are directly proportional to the linear velocity of two of their points having the same radius, and inversely proportional to radii of any two of their points having the same linear velocity.

In the general case

$$\text{Angular velocity} = \frac{\text{linear veloc.}}{\text{radius.}}$$

This may be expressed by formulas as follows:

Let a = linear velocity of a point A, in a body α

Let b = linear velocity of a point B, in a body β

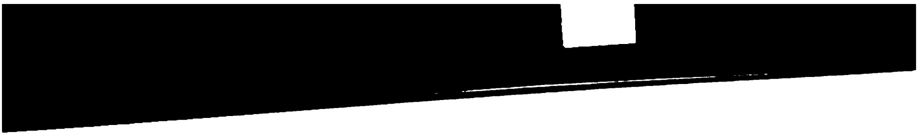
Let r_a = radius of a

Let r_b = radius of b

Let angular velocities be expressed according to the second standard.

$$\text{Angular velocity } \alpha = \frac{a}{r_a}$$

$$\text{Angular velocity } \beta = \frac{b}{r_b}$$



$$\text{If } r_a = r_b \therefore \frac{\text{angular velocity } \alpha}{\text{angular velocity } \beta} = \frac{a}{b}$$

$$\text{If } a = b \ \frac{\text{angular velocity } \alpha}{\text{angular velocity } \beta} = \frac{r_b}{r_a}$$

$$\text{Or, generally, } \frac{\text{angular velocity } \alpha}{\text{angular velocity } \beta} = \frac{a\, r_b}{b\, r_a}.$$

From these relations, the angular velocity of any link in a mechanism can be determined, that of any other being given; or rather the relative angular velocities of any two links may be determined.

In Fig. 43 suppose that d is the fixed link, and that the angular velδcity of a relative to d is given, and that of b is required. a and b have a common point O_{ab} and this of course must have the same linear velocity in each relatively to d. The angular velocities then of a and b are inversely proportional to the virtual radii in each of them, of the point O_{ab}; or,

$$\frac{\text{angular velocity of } b}{\text{angular velocity of } a} = \frac{O_{ab}\, O_{ad}}{O_{ab}\, O_{bd}}.$$

This may be solved graphically. Draw in any direction through O_{ab}, a line representing on any scale the angular velocity of a, $O_{ab}A$; join A to virtual centre O_{bd}, and draw $O_{ad}B$ parallel to it; $O_{ab}B$ now represents the angular velocity of b on the same scale as $O_{ab}A$ represents that of a. This results of course from similar triangles.

In Fig. 44 the angular velocity of a is given; that of c is required, d being the fixed link.

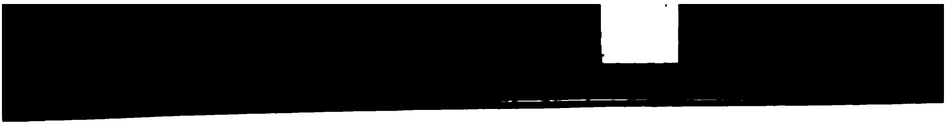
c and a have a common point, their virtual centre relatively to each other; O_{ac}. This point can have but one linear velocity, therefore the

$$\frac{\text{angular velocity } c}{\text{angular velocity } a} = \frac{O_{ac}\, O_{ad}}{O_{ac}\, O_{cd}}.$$

To determine this graphically, from O_{ad} draw a line $O_{ad}A$ in any direction and join O_{ac} to A and draw $O_{cd}B$ parallel to $O_{ad}A$. Then

$$\frac{\text{angular velocity of } c}{\text{angular velocity of } a} = \frac{O_{ad}\, A}{O_{cd}\, B}.$$

Sometimes as in the case of linear velocity, one of the needed virtual centres is inaccessible, and a modification of the solution becomes necessary.



In Fig. 45, suppose that O_{bd} is inaccessible, and the relative angular velocity of *a* and *b* is required. The virtual centre of *a* and *b* is O_{ab}, and it is a point common to the two links, and can have but one linear velocity as a point in either link.

The ratio of angular velocities of the links *a* and *b* therefore, equals the inverse ratio of the virtual radii of the point O_{ab}, as a point in *a* and *b*, $= \frac{O_{ab}\,O_{ad}}{O_{ab}\,O_{bd}}$.

Through O_{ad} draw a line parallel to c. The triangles $O_{ab}\,O_{bd}\,O_{bc}$ and $O_{ab}\,O_{ad}B$ are similar and therefore

$$\frac{O_{ab}\,B}{O_{ab}\,O_{bc}} = \frac{O_{ab}\,O_{ad}}{O_{ab}\,O_{bd}} = \frac{\text{angular velocity of b}}{\text{angular velocity of a}}.$$

If the angular velocity of either link be given, = to any line, a simple graphical construction will determine that of the other.

Where the virtual centre of the two links under consideration, is inaccessible, a different construction may be used. See Fig. 46. O_{ac} is inaccessible and the relative angular velocity of *a* and *c* is required. Join O_{ab} and O_{cd} and draw a line *AB* parallel to *d*, in any convenient location.

$$\text{Now,}\quad \frac{AE}{DE} = \frac{O_{ac}\,O_{ad}}{O_{ac}\,O_{cd}}.$$

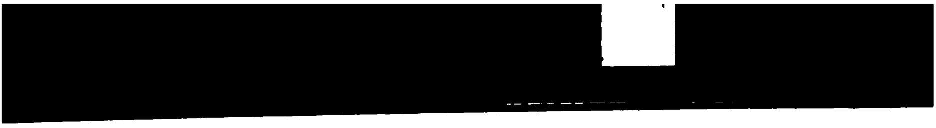
But the ratio of angular velocities of *a* and *c* is = to the inverse ratio of the virtual radii of the point O_{ac} as a point in *a* and *c*, and therefore,

$$\frac{\text{angular velocity of a}}{\text{angular velocity of c}} = \frac{O_{ac}\,O_{cd}}{O_{ac}\,O_{ad}} = \frac{DE}{AE}.$$

The method of determining the relative linear velocities of points in mechanisms has been studied, and also the method of determining the relative augular velocities of links.

In practice it is often necessary to solve these problems for a series of positions; to determine relative velocities throughout a cycle of action.

First, let us take for illustration the slider crank chain, which is used so universally in the steam engine. Suppose that the axis of the crank pin in a steam engine has a given *uniform linear velocity*, and that it is desirable to know the



velocity of the piston at all points of its stroke, relatively to that of the axis of the crank pin.

The velocity of the piston, its rod, and the crosshead is the same since they are rigidly attached and all have a rectilinear reciprocating motion.

The link *c* in the slider crank chain Fig. 47 may therefore represent them, and since all points in *c* have the same velocity any point may be selected as the one whose velocity is to be determined. Let the centre point of the turning pair by which it is attached to the link *b* be taken; this is a point of the link *b*. Also the centre of the crank pin is a point in *b*, and our problem becomes simply, to determine the relative linear velocity of two points in a link; in this case *b*; in the real engine it is the connecting rod.

This problem has been already solved. The velocities of O_{ab} and O_{bc} (Fig. 47) are proportional to their virtual radii or

$$\frac{\text{velocity } O_{ab}}{\text{velocity } O_{bc}} = \frac{O_{ab}\,O_{bd}}{O_{bc}\,O_{bd}}.$$

But if a line be drawn parallel to *b*, it will cut off from the virtual radii, distances proportional to the radii themselves. If then we lay off from O_{ab} along its virtual radius, a distance representing the velocity of O_{ab}, and draw AB through point A, parallal to *b*, then O_{bc} B will represent the velocity of O_{bc}.

Let this be applied to a series of positions.

See Fig. 48.

Draw the circle that is the path of the centre of the crank and divide it into 12 equal parts. Draw another circle concentric with this, the radical distance between the two circles being equal to the constant linear velocity of the crank pin centre. Lay off positions of crosshead pin centre corresponding to A_1A_2 &c., and through these points draw perpendiculars to the centre line of motion. Through B_1B_2 &c. draw lines parallel to the corresponding position of the connecting rod. This cuts off on its corresponding perpendicular a distance equal to the velocity of the crosshead at that instant. So a series of points may be found that locate a curve representing the variations of velocity during the stroke of the piston.

If the piston velocity for each position of the crank, instead of for each of its own positions is required, a diagram may be constructed as follows. (Fig. 49.)

Draw a circle whose radius equals the linear velocity of the crank pin centre. Lay off any number of crank positions; for each of these positions the velocity of the crank pin is the same; therefore the semi-circle represents the crank pin velocity during the stroke. In these radial lines, lay off the corresponding velocities of the piston. Joining the points so determined we get a curve like the one shown. Let these relations of crank and piston velocity followed thoughout a stroke. At first the piston velocity equals o; as shown by the fact, that when the crank is on its centre, the link b (Fig. 50), coincides with the link d, and the virtual centre O_{bd}, coincides with the centre of the cross head pin, or O_{bc}. Therefore the virtual radius of O_{bc} equals o, and its velocity equals o since they are proportional to each other. As the crank moves up from its centre the angle β is greater than α, and therefore AO_{ab} is greater than AO_{bc}, and therefore velocity of crank is greater than velocity of cross head. At some point, however, the connecting rod gets to be at right angles to AO_{ab} and therefore AO_{bc} is the hypothenuse of a right angled triangle, AO_{ab} and the connecting rod being the other two sides. Obviously the virtual radius of O_{bc} is greater that that of O_{ab}, and hence at this instant the velocity of the piston is greater than velocity of the crank. At some instant previous to this the velocities must have been equal, and of course that was when AO_{bc} equaled AO_{ab}. This corresponds to the point M in the diagram of velocities. When the crank reaches a position at right angles to the line of motion, or the link d, the virtual radii of O_{ab} and O_{bc} are parallel, and infinitely long; and therefore equal, and the velocities are equal. This corresponds to A in the diagram.

A similar velocity diagram may be constructed for the "slotted cross head" mechanism.

The constant linear velocity of the centre of the crank pin is given, and the ratio of this, to the linear velocity of any point of C is required.

Consider O_{ab} (Fig. 26) as a point in the link a. a and c have a common point O_{ac}. As a point in a O_{ac} rotates about O_{ad}; and

$$\frac{\text{velocity of } O_{ab}}{\text{velocity of } O_{ac}} = \frac{O_{ad}\,O_{ab}}{O_{ad}\,O_{ac}}.$$

As a point in c O_{ae} rotates about a point O_{cd}, at infinity ; and all points in c have the same velocity as O_{ae}, therefore if the length of the link a equals given velocity of crank pin centre ; O_{ad} O_{ae} will represent velocity in any point in c. Or for construction lay off k on the line of a equal to the velocity of O_{ab} on some scale ; then by similar triangles e will equal the velocity of c. This is true for any position.

Draw the crank circle (Fig. 51) and another circle whose radial distance from the crank circle is equal to the constant linear velocity of A, the centre of the crank pin. CD will then be the velocity corresponding of the link c or the slider.

A series of positions may now be taken and a velocity curve plotted as in the last example. The curve in this case is symmetrical about a vertical line drawn through the centre position of the cross head's travel. Looking at the triangle ABE, we see that the crank travels from the dead centre up in the direction of the arrow the side BE never becomes greater than AB and therefore, the velocity of cross head never becomes greater than that of A, but becomes equal to it when crank is at right angles to the centre line of motion, or the link d.

In the polar diagram, in this case, the curves of cross-head velocity become circles, whose diameter equals the radius of the circle that represents the constant linear velocity of the centre of the crank pin.

It will now be shown how the "Whitworth Quick Return" motion, is developed from the slider crank chain.

Suppose that in Fig. 52 the link b has a constant angular velocity. It will then communicate to d a variable angular velocity. Because the lever arm of b upon d is variable. This variable angular velocity of d may be determined and plotted on a diagram. The problem is to find the angular velocity of d that of b being given. b and d have a common point, their virtual centre O_{bd} ; and this has the same linear velocity whether in b or d, and therefore the angular velocities are proportional to the virtual radii of that point relatively to the fixed link, in this case a. Then drawing AB in any direction representing the angular velocity of b, given, from similar triangles it is seen that angular velocity of $d = CD$. This may be done for a series of positions and a diagram of velocities drawn. Fig. 53.

d rotates about *A* with a varying angular velocity as shown by the curve in the velocity diagram. Since linear velocity of any point of constant radius is proportional to angular velocity, therefore the linear velocity of *E* could be represented by the same curve. Suppose now *AE* be made the crank of another slider crank chain. The relation of velocity of crank pin axis and slider will be different from what it was when the linear velocity of the crank pin centre was constant. This may be determined by the application of the principles already considered.

Draw a circle Fig. 54 the path of the crank pin centre divide into equal parts and lay off outside of it the varying velocities. Lay off cross head positions as in the other problem. The diagram of velocities now shows that if rotation be in the direction of the arrow, the return stroke of the slider is at a higher velocity than the forward stroke. It is therefore a "quick return motion." If now the slider be made into the ram, carrying the tool post of a shaping machine, the forward stroke, when the tool is cutting will be slow, and the return stroke, when the tool is idle will be fast; this is the desired result.

Another quick return motion may be derived from a slider crank chain in which the line of the slider's travel does not pass through the centre of rotation of the crank.

Let *AB* Fig. 55, be the line of travel of the slider, and the smaller circle, the path of the centre of the crank pin. Let *B* be the right hand extreme of the sliders travel. Draw through *B* and O a straight line. *BC* will then be the connecting rod. To *BC* add the radius of the crank circle and with this distance as a radius and O as a centre, strike the arc cutting *AB* at *A*. Join *A*O. *AD* is then the other extreme position of the connecting rod. Divide the arc *CED* into any convenient number of equal parts. Also the arc *DFC*. Draw the outside circle making *CA* = velocity (constant) of centre of crank pin. Let O*H* be one of the crank positions. *K* will be the corresponding slider positions and *HK* will be the corresponding connecting rod position. Draw *MN* parallel to *HK*; then *NK* will be the velocity of the slider on the same scale that *HM* is the velocity of crank pin centre. Carrying out this construction for all the as-

sumed positions we get a velocity curve as shown. If rotation occur in the direction of the arrow the slider will move from A to B slowly, and return from B to A quickly. This mechanism is applied to planing machines.

Still another quick return may be considered. Connect a four link mechanism as follows, Fig. 56, fix d; join to it a crank a by a turning pair; then to this join b by a turning pair; but join b and c by a sliding pair and then join c to d by a turning pair.

If now a be given a constant angular velocity about O_{ad} it will communicate to c a vibratory motion with varying angular velocity.

In Fig. 57 consider the position of links O_{ad} O_{ad} O_{cd}. O_{ab} is supposed to have a constant linear velocity and the velocity of E' in the link c is required. O_{ab} is a point in a and E' is a point in c. The virtual centre of these two links is O_{ac}; draw BA equal linear velocity of O_{ab}; draw $A\,O_{ad}$ then $O_{ac}\,C$ is the linear velocity of O_{ac} on same scale. Join C and O_{cd}, draw EF parallel to AB then FE equal velocity of E' on same scale. The same method gives us the values in the other positions, and if a curve of velocities be plotted it would be found that if rotation takes place with the arrow, E' will move toward the right quickly and return slowly.

This mechanism has been used in shapers.

PARALLEL ON STRAIGHT LINE MOTIONS FROM LINKWORK.

Usually in machinery if it is desired to constrain rectilinear motion, prismatic guides are used. But sometimes it is necessary, or convenient, to accomplish the same result by means of linkwork. As a preliminary to this study of parallel motions, let us consider some special properties of the simple linkwork parallelogram *i. e.* a four link chain, with its opposite links equal, joined by turning pairs.

Let Fig. 58 represent such a chain with the point O fixed, instead of an entire link. Draw through O, a line OBC cutting b and c in B and C. In whatever position the mechanism be placed the points O, B and C will remain in the same straight line.

$$\text{For } \frac{DB}{EO} = \frac{DC}{EC} \quad \therefore \quad DB = EO \cdot \frac{DC}{EC}$$

but all of the terms in the second member of this

are constant; therefore DB must be constant, and therefore B always has the same position on the link b.

$$\text{Also, } \frac{OC}{OB} = \frac{EC}{ED}.$$

Therefore, relation of motion of B and C is always expressed by the ratio $\frac{EC}{ED}$, and since the ratio is necessarily constant, it will be seen that whatever curve or figure C describes, B will describe the same curve or figure on a scale proportional to the ratio $\frac{ED}{EC}$.

Also, if C be fixed, and O free, its movements will be reproduced at B on a reduced scale represented by the ratio $\frac{EC}{DC}$.

This linkage, called a pantograph is often used for reduction of drawings; and also for connecting an indicator drum to the crosshead of an engine to be tested.

In the linkage, Fig; 59, AB and CD are equal and, in the position shown, parallel. B and C are fixed points. AD connects the links AB and CD. O is the middle point of AD. If the linkage be moved upward, A will be drawn toward the right a certain amount, and D will be drawn toward the left an equal amount, and the point O will move vertically. *Within limits* this linkage gives a very close approximation to a straight line motion. It is used in the Richards Steam Engine Indicator, for the multiplying of the piston motion at the pencil, and carrying the latter in a vertical straight line. This linkage was also used, in combination with the pantograph, by James Watt, for the guiding of the upper end of the piston rod of his engine, in a straight vertical line.

In Fig. 60, let OB be the walking beam of a Watt engine, and BC a link that joins it to the top end of the piston rod at C. It is now required that *C* shall move in a straight vertical line. Draw CE parallel to BA and AE parallel to [illegible]. Join C to O. ABCE is a parallelogram linkage, and [illegible] be guided in a straight vertical line, *C* will reproduce [illegible]motion on an enlarged scale, or, it will be guided as [illegible]ed.

The point D is guided by making DF=AD and connecting it at F to a link FG=AO rotating about G. This is equivalent to guiding the point D by means of the linkage shown in Fig. 59.

BC might just as well have been extended an amount equal BC, and attached to a link equal BO ; but the other construction is adopted because of the desirability of compactness.

It is required to design a parallel motion like that in Fig. 59. It may be done as follows :

Let AB, Fig. 61, be the path of the point ; O the centre of the link ; M the mid-position of the point ; and NP equal travel of M. It is desirable that the end of the link that vibrates about O shall move equal distances, to the right and to the left of AB during its vibration. Lay off from A downward on the point path a distance equal to ¼ NP. Join D and O. Draw FE at right angles to OD ; then OF will be the required length of the link. Join FM. If now M is to be in the centre of the connecting link, then make MG equall FM and GO will equal OF, and be parallel to it. But if M is to be above the middle, lay off MH in the proper proportion, and draw HO″ parallel to OF ; from B lay off upward a distance equal to AM, and through O and M′ draw a straight line. OH will then be the proper length of link.

In Fig. 62 as *a* vibrates about O_{ad}, B describes a straight vertical line. Obd is the vertical centre of *b* relative to fixed link *d*. Therefore B for the instant is moving at right angles to Obd. But as B moves down the vertical centre Obd moves down also at same same rate, and therefore the line B Obd, which is the vertical radius of B remains parallel to *d*, and B always moving at right angles to it, moves always at right angles to *d*, or along a vertical straight line.

Another exact straight line motion is that invented by M. Peaucillier. It consists of eight links arranged as shown in Fig. 63.

a, b, c, and d are equal and forma rhombus in all positions ; e and f are also a pair of equal links, jointed as shown, as are also h and g.

Let us *consider first* six links arranged as in Fig. 64.

From the equality and arrangement of links P, N and M are in the same straight line. Then $PS^2 = PV^2 + VS^2$ also $SM^2 = VM^2 + VS^2$. Subtracting second from first, we have $PS^2 - SM^2 = PV^2 - VM^2 = (PV + VM)\ (PV - VM) = PN \times PM$. But for any mechanism $PS^2 - SM^2$ equals a constant. Therefore $PN \times PM$ also equals a constant, and this is true for any position of the mechanism.

Consider now the complete mechanism, Fig. 65.

It is known that $PN \times PM = PN' \times PM'$; or $\frac{PN}{PN'} = \frac{PM'}{PM}$.

The triangles PNN′ and PMM′, are similar and the angle PNN′ equals the angle PM′M. But the points N′N and P lie in the circumference of a circle, and the angle PNN′ is therefore an angle on a semi-circle, and therefore equal to a right angle, and hence PM′M equals a right angle, or M is in a line drawn through M′ at right angles to the centre line of the mechanism, and, since this is true for all positions, therefore M, as the mechanism is moved, will describe a straight line at right angles to centre line of motion.

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GEARS OR TOOTHED WHEELS.

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The surface contact of turning or sliding pairs, is sometimes replaced in machines by elements having line contact, or by "higher pairs," as in the case of "cams," etc. The most common example, however, is gearing, or toothed wheels.

The simplest toothed wheel mechanism is shown in Fig. 66. It is a chain of three links, a, b, and c. c is connected to a by a turning pair, as is also b; but the connection between b and c, is by means of wheel teeth, which form a higher pair of kinematic elements, since they have line contact only; b and c are called "spur gears."

There will be three virtual centres in this mechanism: O_{ab}, O_{ac}, and O_{bc}; and these will be, of course, in the same straight line. O_{ac} is the centre of the turning pair connecting a and c; O_{ab} is the centre of the turning pair connecting a and b; therefore O_{bc}, as well as O_{ab} and O_{ac}, must be somewhere in the line of centres. See Fig. 67. b is a body that rotates about O_{ab} with a constant angular velocity; c is a body that rotates about O_{ac} with a constant angular velocity; their angular velocity ratio must therefore be constant. But spur gears are bodies that rotate about fixed centres with a constant angular velocity ratio. Therefore b and c may represent a pair of spur gears, whose virtual centre is to be found. Let the constant velocity ratio be given

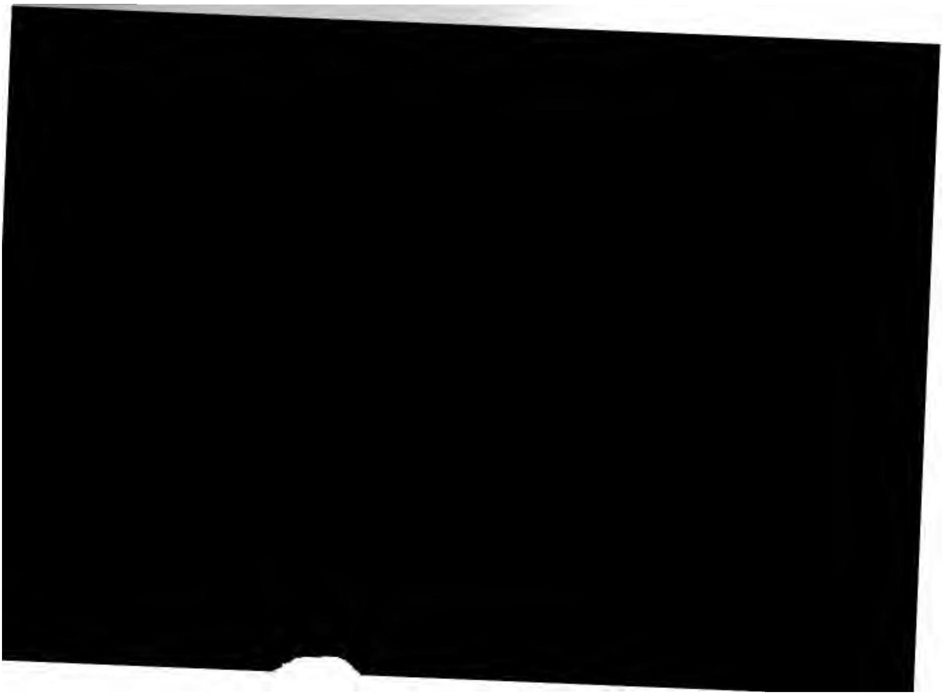
$$= \frac{m}{n} = \frac{\text{angular velocity b}}{\text{angular velocity c}}.$$

O_{bc} is a point common to b and c; it may be considered as a point of b or c. As a point in b, it must have the same linear velocity that it has as a point of c. Since there are two points in a mechanism, (O_{bc} in b. and O_{bc} in c,) that have the same linear velocity; therefore, their angular velocities must be inversely proportional to their virtual radii. But the virtual radii are $O_{bc}\,O_{ab}$, and $O_{bc}\,O_{ac}$ and the angular velocity ratio is given $= \frac{m}{n}$. Therefore, $\frac{O_{bc}\,O_{ab}}{O_{bc}\,O_{ac}} = \frac{m}{n}$; but O_{bc} is on the line of centres, and its distance from O_{ab} is

to its distance from O_{ac} as m is to n. Or the virtual centre O_{bc} is a point that divides the line of centres into parts that are to each other inversely as the angular velocity ratio.

To determine O_{bc} by construction, let m = the line x y, on some scale, and let n = the line x′ y′ on the same scale. From O_{ab} in any direction as $O_{ab}A$, lay off a line = n; from O_{ac}, parallel to $O_{ab}A$, but on the opposite side of the line of centres, lay off $O_{ac}B = m$; join A and B, and the point in which this line cuts the line of centres will be O_{bc}; because $\frac{O_{ab}\,A}{O_{ac}\,B} = \frac{O_{ab}\,O_{bc}}{O_{ac}\,O_{bc}} = \frac{m}{n}$. Therefore, the line AB divides the line of centres into parts that are inversely proportional to the angular velocity ratio, and so locates O_{bc}.

As these bodies b and c rotate, different points become in turn the virtual centre O_{bc}; but since this is always at a constant distance from the centres of rotation of b and c, therefore, the centrode of O_{bc} in b is a circle, with O_{ab} as a centre; and $O_{ab}\,O_{bc}$ as a radius; and the centrode of c is a circle, with O_{ac} as a centre, and $O_{ac}\,O_{bc}$ as a radius. These centrodes correspond to what is technically called *pitch circles* in gears. As the bodies rotate with the given angular velocity these centrodes roll upon each other. It has already been seen that in plane motion the relative motion of two bodies is conditioned by the rolling on each other of two curves, the centrodes of each relatively to the other; that as long as these centrodes remain unchanged, the relative motion remains unchanged; and conversely, as long as the relative motion remains unchanged, the centrodes remain unchanged. If the motion be given, the centrodes may be determined; and if the centrodes be given, the motion may be determined. Usually the centrodes are so complex that they are of little practical use in the determination of motion. In this case it is different however, for the centrodes are circles whose radii and centres are easily determined. In the figure the centrodes are the circles and as long as they roll on each other without slipping, the relative motion, or velocity ratio remains absolutely unchanged. Passing now from plane figures to bodies, the



circles become cylinders, and O_{bc} becomes a line of contact. If these cylinders roll on each other without slipping, motion will be communicated from one to the other with a constant velocity ratio; for, all points in the surface of the cylinder will have the same linear velocity, therefore their angular velocities will be proportional inversely to the radii. Friction gears are sometimes made in this way; but the difficulty in preventing slipping, makes them undesirable where it is necessary to maintain the velocity ratio absolutely. Therefore in practice, teeth are formed on the pitch cylinders, of sufficient size and strength, so that one wheel is compelled to move when the other moves.

If the same relative motion is to be transmitted as before, the form of the teeth must be such as to correspond to the same centrodes as before. These centrodes are so simple that they may be used directly in the determination of proper tooth forms.

Teeth of almost any form may be used, and it is insured that the *average* velocity will be right. By the use of teeth that are not of the proper form the velocity is continually varying between the maximum and minimum values, and while these as variations of velocity are often not important, yet they are the cause of noise and unnecessary wear. The formation of correct tooth forms, therefore, for the transmission of a constant velocity ratio, is not alone a matter of theoretical importance but one also that is practically worth cansidering.

The essential condition to be fulfilled is *that at whatvver point there may be contact between the teeth, the velocity ratio between the wheels must remain unchanged.* But for a given velocity ratio, there is a fixed virtual centre for the two wheels; the condition may therefore be expressed, by saying; *at whatever point there may be contact between the teeth their virtual centre must remain unchanged.* The only motion that one tooth can have relatively to another at their line of contact, is one of sliding. Thus, Fig. 68, the teeth A and B are in contact at a and the only possible motion of A relatively to B, is one of sliding; the direction of this motion

would be then, along a tangent CD to the curves at the point of contact. The virtual centre must be in a normal to this tangent. But since the virtual centre is always known (the velocity ratio being given), we can always draw the tangent to the tooth curves for any given position of the point of contact.

Thus let A, Fig. 69, be any position of the point of contact; join O_{bc} to this point, and draw a line at right angles to it through A; tooth curves touching at A in such a way that this line DE is thus tangent, will fulfill for the instant, the condition of transmitting motion with the required velocity ratio. On the other hand a normal to the tangent to the curves in contact at B cuts the centre line of motion at another point and while these tooth curves would serve to transmit motion they would not transmit motion with required velocity ratio, as they correspond to different centrodes.

Having given any curve that will serve for a tooth outline in one gear, the corresponding curve may be found in the other gear, that shall engage with the given curve for the transmission of a constant velocity ratio.

Let $n \div m$ be the given velocity ratio. Draw the line of centres AB, Fig. 70. Let P be the "pitch point," *i. e.*, the point of contact of the pitch circles, or the virtual centre of the two gears. To the right from P lay off a distance $PB = m$; from P toward the left lay off $PA = n$. A and B will then be the centres of the wheels required, and the pitch circles may be drawn through P. Let a b c, be any given curve in the wheel A. It is required to find the curve in B that shall engage with a, b, c, to transmit the constant velocity ratio required between the wheels. A normal to the point of contact must pass through the virtual centre. If, therefore, any point as a, be taken in the given curve, and a normal to the curve at that point be drawn as $a\alpha$; then when a is the point of contact, α will coincide with P. Also, if $c\gamma$ is a normal to the curve at c, then γ will coincide with P when c is the point of contact between the gears, and since b is in the pitch line it will it-

self coincide with P when it is the point of contact. Suppose now that A and B are discs of card-board, and that A overlaps B, and that a thread is stretched so as to indicate the centre line AB. Suppose also that they can be rotated so that the pitch circles roll on each other without slipping. Roll a down till α reaches P, and prick a through upon B; then make b coincide with P, and prick it through; then make γ coincide with P, and prick c through. This will give these points in the required curve in B, and through these the curve may be drawn. The curve could, of course, be more accurately located by using more points. These points that locate the curve in B, might be determined geometrically.

Many curves could be drawn that would not serve for tooth outlines; but if any curve that will serve be given, the corresponding curve may be easily found. There would be, therefore, almost an infinite number of curves, that would fulfill the requirements of correct tooth outlines. But in practice two kinds of curves are found so convenient that they are almost exclusively used. They are cycloidal and involute curves.

CYCLOIDAL CURVES FOR GEAR TOOTH OUTLINES.

In Fig. 71, let b and c be the pitch circles of a pair of wheels always in contact at O_{bc}. Also let m be the describing circle in contact with both at the same point. M is the describing point.

When one curve rolls upon another, the virtual centre of their relative motion is always their point of contact. For, as the motion of rolling excludes slipping, the two bodies must be stationary relatively to each other at their point of contact, and bodies that move relatively to each other, can have but one such stationary point in common, and that is their virtual centre.

When therefore m rolls in or upon b or c, its virtual centre relatively to either is their point of contact. The point M therefore must describe curves whose direction at any point is at right angles to a line joining that point to O_{bm} or

O_{cm}, according as it rolls upon b or c. Suppose now the two circles b and c to revolve about their centres, and to roll upon each other, at the same time being always in contact at O_{bc}. And also suppose m to roll at the same time upon both curves, the three circles being always in contact at the one point. The point M will then describe simultaneously a curve b_1 on b and c_1 on c. Since M describes the curves simultaneously, it will always be the point of contact between them in any position. And since the point M moves always at right angles to a line which joins it to O_{bc}, therefore the normal to the tooth surfaces at their point of contact will always pass through O_{bc}, and the condition for constant velocity ratio transmission is fulfilled. But these curves are precisely the epicycloid and the hypocycloid that would be drawn by M in the generating circle, by rolling on the inside of c and the outside of b. Obviously, then, the epicycloids and hypocloids generated in this way, used as tooth profiles will transmit a constant velocity ratio.

This proof is independent of the size of the generating circle; its diameter may therefore be equal to the radius of b. But the hypocycloids generated by rolling within b would be a straight line, coinciding with the diameter of b. Clearly, in this case the profiles of the teeth of b become radial lines; and therefore the teeth are thinner at the base than at the pitch line; for this reason they are weaker than if a smaller generating circle had been used. All tooth curves generated with the same generating circle will work together, the pitch being the same. It is therefore necessary to use the same generating circle for a set of gears that need to interchange.

The describing circle may be made still larger. In the first case the curves described have their convexity in the same direction, *i. e.*, they lie on the same side of a common tangent. When the describing circle is made equal radius of b, then one curve becomes a straight line tangent to the other curve. As the describing circle becomes still larger, the curves have their convexity in opposite direction. As the circle approximates equality with b, the hypocycloid

grows shorter, and finally, when the describing circle equals b, it becomes a point, which is the generating point in b, which is now the generating circle. If this point could be replaced by a pin so small as to have no sensible diameter, it would engage with the epicycloid generated by it to transmit constant velocity ratio.

But a pin of no sensible diameter will not serve as a wheel tooth, and so a proper diameter must be assumed, and a new curve be laid off to engage with it in the other gear.

In Fig. 72, AB is the epicycloid generated by a point in the circumference of the other pitch circle. CD is the new curve that is drawn tangent to a series of positions of the pin as shown. The pin will engage with this curve, CD, and transmit the constant velocity ratio as required.

In Fig. 71, let it be supposed that when the three circles rotate constantly tangent to each other at the pitch point O_{bc}, a pencil is fastened at the point M in the circumference of the describing circle. If this pencil be supposed to mark simultaneously upon the planes of b, c and the paper, it will describe on b an epicycloid, on c a hypocycloid, and on the paper an arc of the describing circle. Since M is always the point of contact of the cycloidal curves (because it generates them simultaneously), therefore, in cycloidal gear teeth, *the locus or path of the point of contact is an arc of the describing circle.*

In the cases already considered, where an epicycloid in one wheel engages with a hypocycloid in the other, the contact of the teeth with each other is all on one side of the line of centres. Thus, in Fig. 71, if the motion be reversed the curves will be in contact until M returns to O_{bc} along the arc MDO_{bc}, but after M passes O_{bc}, contact will cease. If c were the driving wheel, the point of contact would approach the line of centres; if b were the driving wheel, the point of contact would recede from the line of centers. Experience shows that the latter gives smoother running because of better conditions as regards the friction between the tooth surfaces. It would be desirable therefore that the wheel with the epicycloid tooth curves should always be the driver.

But it is more convenient if either wheel may be used as driver, in order that the varying conditions in practice may be met.

There is another reason why contact should not be all on one side of the line of centres, which may be explained as follows:

[Definitions. The angle through which a gear wheel turns while one of its teeth is in contact with the corresponding tooth in the other gear, is called *the angle of action*. The arc of the pitch circle corresponding to the angle of action is called *the arc of action*.]

The arc of action must be greater than the "pitch arc," (the arc of the pitch circle that includes one tooth and one space), or else one tooth will cease to have contact before the contact begins between the next pair of teeth; and the constrainment would not be complete. Thus, in Fig. 73, let AB and CD be the pitch circles of a pair of gears, and E the describing circle. Let an arc of action be laid off on each of the circles from P, as Pa, Pc and Pe. Through e, about the centre O, draw an addendum circle; *i. e.*, the circle that is the limit of the points of the teeth. Since the circle E is the path of the point of contact, and since the addendum circle limits the points of the teeth, their intersection e is the point at which contact ceases, rotation being as indicated by the arrow. If the pitch arc is just equal to the assumed arc of action, contact will be just beginning at P when it is ceasing at e; but if the pitch arc be greater than the arc of action, contact will not begin at P till after it has ceased at e, and there will therefore be an interval when AB will not drive CD.

The greater the arc of action, the greater the distance of e from P, on the circumference of the describing circle. The direction of pressure between the teeth, is always a normal to the tooth surface, and this always passes through the pitch point; therefore the greater the arc of action; *i. e.*, the greater the distance of e from P, the greater the obliquity of line of pressure. The pressure may be resolved into two components one at right angles to the lines

of centres and the other parallel to it. The first is resisted by the teeth of the follower wheel, and therefore produce rotation ; the second is resisted at the journal, and so produce friction.

From this it will be seen that the greater the arc of action, the greater will be the average obliquity of the line of pressure, and therefore the greater the component of the pressure that produces wasteful friction. If it can be arranged so that the arc of action shall be partly on each side of the line of centres, ¡then the arc of action may be made greater than the pitch arc, (usually = about 1½ times the pitch arc) and still the obliquity of the prssure line may be kept within reasonable limits, contact between the teeth will be insured in all positions, and either wheel may be the driver.

This result is accomplished by using two describing circles as in Fig. 74. Suppose the four circles A, B, α and β to roll constantly tangent at P. α will describe an epicycloid on the plane of B, and a hypocyloid on the plane of A, and these curves will engage with each other to drive correctly. β will describe an epicycloid on A and a hypocycloid on B, and these curves will also engage to drive correctly. If the epi- and hypocycloid in each gear be drawn through the same point on the pitch circle, a double curve tooth outline will be located and one curve will engage on one side of the line of centres, and the other on the other side. If A is the driver in the sense indicated by the arrow, contact will begin at D and the point of contact will follow an arc of α to P, and then an arc of β to C.

INVOLUTE TOOTH CURVES.

An involute is a curve generated by a point in a straight line that rolls without slipping, upon the circumference of a circle. If a string be wound around a cylinder, and a pencil point attached to its end, this point will trace an involute as the string is unwound from the cylinder. Or, if the point be constrained to follow a tangent to the cylinder,

and the string be unwound by rotating the cylinder about its axis, the point will trace an involute on a plane that rotates with the cylinder.

To illustrate, let α, Fig. 75, be a circular piece of wood, free to rotate about c as a centre; β is a circular piece of card board made fast to α; AB is a straight edge that is held stationary with its edge tangent to α. A string is wound on the circumference of α, and has a pencil point at its end A. As A moves along the straight edge to B, α and β rotate about C, and A traces an involute DB upon β. The relative motion of the tracing point A and of β, being just the same as if the string had been simply unwound from stationary α.

[PROOF.—Give to both β and the tracing point a motion equal and opposite to that which β had while A moved to B. D will then return to A, and B will move to B′; *i. e.*, through an equal angular distance. It will be seen, since β is the same as if it had had no motion, that the motion of A relatively to β is from A to B′, and from its method of attachment its path would be an involute of the circle α, through A and B′. Thererore it is evident that A would describe upon β an involute DB.]

In Fig. 75, if the tracing point be at B, and if it be caused to return along the straight edge to A, the point will follow the involute BD.

The virtual centre of the tracing point is always the point of tangency of the string with the cylinder, and therefore the string, or the straight edge in Fig. 75, is always at right angles to the direction of motion of the tracing point, and hence is always a normal to the involute curve.

Let α and β, Fig. 76, be two base cylinders, and let AB be a cord wound upon α and β, and passing through the virtual centre P, which corresponds to the required velocity ratio. Let α and β be supposed to rotate so that the cord is wound up on α and off from β. Then any point c in the cord, will move from A toward B, and, if it be a tracing point, will trace an involute of B, on the plane of β (extended beyond the base cylinder) and also will trace an involute of

a_1 upon the plane of a. These two involutes will serve for tooth profiles for the transmission of the required velocity ratio, because AB is the constant normal to both curves, at their point of contact, and it passes through P the virtual centre that corresponds to the required velocity ratio. And so the necessary condition is fulfilled.

Since a point in the line AB describes the involute curves simultaneously, the point of contact of the curves is always in the line AB, or AB is the path of the point of contact.

One of the advantages of involute curves for tooth profiles, is that a change in the distance between centres of the gears, does not interfere with the transmission of a constant velocity ratio. This may be proved as follows. In Fig. 76 it is seen from similar triangles that $\frac{OB}{O'A} = \frac{OP}{O'P}$; that is, the ratio of the radii of the base circles, is equal to the ratio of the radii of the pitch circles, and this of course equals the inverse ratio of angular velocities of the gears. Suppose now that O and O′ be moved nearer together, the pitch circles will be made smaller, but the ratio of their radii must be equal to the unchanged ratio of the radii of the base circles, and therefore the velocity ratio remains unchanged; and also, the involute curves, since they are generated from the same base cylinders, will be the same as before, and therefore with the same tooth outlines, the same, constant velocity ratio will be transmitted as before.

If a pitch circle be divided into as many equal parts as there are required to be teeth in the gear, then the arc included between two of these divisions is the *circular pitch* of the gear. Circular pitch may be also defined as the distance on the pitch circle occupied by a tooth and a space; or otherwise it is the distance on the pitch circle from any point of a tooth to the corresponding point in the next tooth. A fractional tooth is impossible, and therefore the

circular pitch must be such a value, that the pitch ci cumference is divisible by it.

Let P=circular pitch in inches.

Let D=pitch diameter in inches.

Let N=number of teeth.

Then $NP=\pi D$.

$$N=\frac{\pi D}{P} \text{ and } D=\frac{NP}{\pi} \text{ and } P=\frac{\pi D}{N}$$

From these relations we may evidently find any one the three values P, D and N, if the other two are given.

But this relation introduces the inconvenient value of As N must be an integer, and as P is usually taken some convenient part of an inch, it often follows that contains an awkward fraction. This difficulty may overcome by the use of what is known as *diametral pit* Circular pitch is obtained by dividing the circumference the pitch circle by the number of teeth. So also a rat may be obtained by dividing the diameter of pitch circle l number of teeth ; or if p equal the diametral pitch, then would equal $\frac{D}{N}$. In practice, however, it is found mo convenient to invert this ratio. Thus $p=\frac{N}{D}$; or p equ the number of teeth per inch of pitch circle diameter.

We have $P=\frac{\pi D}{N}$ and $p=\frac{N}{D}$ from which it follows th $Pp=\frac{\pi D}{N}\times\frac{N}{D}=\pi$ or π is always equal to the product the circular and diametral pitch. From which it is cle that either P or p may be easily found, the other bei given. In this system, since p is a whole number, P w be the quantity containing the inconvenient fraction.

So if circular pitch is used, the pitch diameter is an i convenient quantity, and if diametral pitch is used the c cular pitch is an inconvenient quantity. In making pa terns for large cast gears, or in laying out the wooden tee of mortice gears, it is more essential that the circular pi

[redacted]

[redacted]

should be a convenient value than that the pitch diameter should be; therefore for this kind of work, circular pitch is used. But in small gears that are cut with milling cutters, convenient values for diameter are necessary for sizing the blanks, and, since circular pitch is not in this case "stepped off," but obtained by moving the blank through a certain angle, by means of an index plate, the fraction in the value for circular pitch is not at all an inconvenience.

As a result of these considerations circular pitch is used for large cast, and mortice gears, and diametral pitch for small cut gears.

In Fig. 77, b, e and k are the *pitch points* of the teeth; a b is the *face* of the tooth; b m is the *flank* of the tooth; AD is the total depth of the tooth; AC is the working depth; AB is the *addendum*, and a circle through A is the *addendum* circle. *Clearance* is the excess of total depth over the working depth, = C D. *Backlash* is the width of a space on the pitch line, minus the width of the tooth on the same line. In cast gears whose tooth surfaces are not "tooled," quite an amount of backlash needs to be allowed, because of the unavoidable imperfections in the surfaces. In cut gears, however, it may be reduced almost to zero, and the tooth and space, measured on the pitch circle, may be considered equal.

RACKS.

A rack is a wheel whose pitch radius is infinite; its pitch circle therefore becomes a straight line, and is tangent to the pitch circle of the wheel or pinion with which the rack engages. The line of centres is a normal to the pitch line of the rack, through the centre of the pitch circle of the pinion. The pitch of the rack is determined by rectifying the circular pitch of the engaging wheel and laying it off on the pitch line of the rack. The outline curves of rack teeth, like those of wheels of finite radius, may be generated by a point in the circumference of a circle that rolls on the pitch circle. Since, however, in this case the pitch circle is a straight line, the tooth curves so described will be cycloids,

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[redacted]

both for the flanks and faces. In Fig. 78 AB is the pitch circle of the pinion, and CD is the pitch line of the rack; a and b are describing circles. Suppose, as before, that all move without slipping, so that they remain constantly tangent at P. A point in the circumference of a, will then describe simultaneously a cycloid on CD and a hypocycloid within AB, that will work together as correct tooth profiles; also, a point in the circumference of b will describe a cycloid on CD and an epicycloid on AB, and these will also be correct tooth outlines. The path of the point of contact is easily determined. Draw the addendum circle EF of the pinion, and the addendum line GH of the rack. If the pinion turn clockwise and drive the rack, contact will begin at e and follow arcs of the describing circles, through P to K.

It is obvious that a rack cannot be used where rotation is continuous in one "sense," but only where rotation is reversed.

Involute curves may also be used for the outlines of rack teeth. Let CD and C'D' (Fig. 79) be the pitch lines. When it is required to generate involute curves for tooth outlines, for a pair of gears of finite radius, a line is drawn through the pitch point, at a given angle to the line of centres, (usually 75°); this line is the path of the point that generates the two involutes simultaneously, and therefore the path of the point of contact between the curves; it is also the common tangent to the two base circles, which may now be drawn, and used for the describing of the involutes. To apply this to the case of a rack and pinion: Draw EF, Fig. 79. The base circles must be drawn tangent to this line. AB will therefore be the base circle for the pinion. But the base circle in the rack has an infinite radius, and a circle of infinite radius drawn tangent to EF would be a straight line coincident with EF. Therefore EF is the base circle of the rack. But an involute to a base circle of infinite radius is a straight line normal to the circumference. In this case a straight line perpendicular to EF. Therefore the tooth profiles of a rack in the involute system will always be straight lines perpendicular to

the line tangent to the base circle of the pinion and passing through the pitch point. If, in Fig. 79, the pinion move clockwise, and drive the rack, the contact will begin at E, the intersection of the addendum line of the rack GH, and the base circle AB of the pinion, and will follow the line EF through P, to the point where EF cuts the addendum circle LM of the pinion.

ANNULAR GEARS.

Both of the centres of a pair of gears may be on the same side of the pitch point, and this arrangement corresponds to what is known as an annular gear and pinion. Thus in Fig. 80, AB and CD are the pitch circles and their centres are both above the pitch point P. Teeth may be constructed that shall transmit rotation from AB to CD, or *vice versa*. AB will be an ordinary spur pinion, but it is obvious that CD becomes a ring of metal with teeth on the inside: *i. e.*, an annular gear. In this case α and β may be describing circles, and a point in the circumference of α will describe simultaneously on the planes of AB and CD, hypocycloids, and a point in the circumference of β will describe simultaneously on the planes of AB and CD, epicycloids; and these will engage to transmit a constant velocity ratio.

It will be obvious that the space inside of an annular gear corresponds to a spur gear of the same pitch and pitch diameter, and with tooth curves drawn by the same describing circle.

Let EF and GH (Fig. 80) be the addendum circles. If the pinion move clockwise and drive the annular gear, the path of the point of contact will be from e along the circumference of α to P, and from P along the circumference of β to K.

The construction of involute teeth for an annular gear and pinion, involves exactly the same principles as in the case of a pair of spur gears. The only difference of detail is that the describing point is in the tangent to the base

circles *produced* instead of being between the points of tangency.

Let O and O′ be the centres, and AB and I J the pitch circles of an annular gear and pinion. Through the point of tangency of the pitch circles, P, draw the path of the point of contact, at the given angle with the line of centres. With O and O′ as centres draw tangent circles to this line. These will be the involute base circles. Let the tangent be replaced by a cord, made fast say at K, winding on the circumference of the base circle past D, and then around the base circle FE in the direction of the arrow, then passing over the pulley G which holds it in line with PB. If rotation be supposed to occur with the two pitch circles always tangent at P without slipping, any point in the cord beyond P toward G will describe an involute on the plane of I J, and another on the plane of AB, and these two will be the correct involute tooth profiles requires.

Draw NQ and LM, the addendum circles. Then if the pinion move clockwise and drive the annular gear, the point of contact starts from e, and moves along the line GH through P to K.

In case of a pair of spur gears meshing, as in Fig. 66, the sense of motion is reversed. In the case of an annular gear and pinion, the sense of motion is the same.

INTERCHANGEABLE SETS OF GEARS.

In practice it is desirable to have what is called "interchangeable sets" of gears; *i. e.*, sets in which any gear will "mesh" correctly with any other, from the smallest pinion to the rack, and in which, except for limiting conditions of size, any spur gear will mesh with any annular gear. Interchangable sets may be made in either the cycloidal or involute system. A necessary condition in any such set is, that the pitch shall be constant; because the thickness of tooth on the pitch line must always equal the width of the space (less the clearance, if there is any,), and if this condition is unfulfilled the gears cannot engage properly, no matter what the form of the tooth outlines.

The second condition for an interchangable set, in the cycloidal system, is that the size of the describing circle shall be constant. If the diameter of the describing circle equal the radius of the smallest pinion's pitch circle, the flanks of this pinion will be radial lines, and the tooth will therefore be thinner at the base than at the pitch line. As the gears increase in size with this constant size of generating circle, the teeth grow thicker at the base, and so the weakest teeth are those of the smallest pinion.

It is found unadvisable to make a pinion with less than 12 teeth; and also that, if the radius of a 15-tooth pinion be selected for the diameter of the describing circle, the flanks in a 12-tooth pinion will be very nearly parallel, and may therefore be cut with a milling cutter; which would not be possible if the describing circle were made larger, causing the space to become wider at the bottom than at the pitch circle. Therefore the describing circle whose diameter equals the pitch radius of a 15-tooth pinion is the maximum possible describing circle for milled gears, and it is the one usually selected. With each change in the number of teeth, at constant pitch, the size of the pitch circle changes; and so the form of the tooth outline, generated by a describing circle of constant diameter, also changes. From which it would seem that for any pitch, a separate cutter would be required for every possible number of teeth. Practically, however, this is not necessary. The change in

the form of tooth outline is much greater in a small gear, for any increase in the number of teeth, than in a large one. And it is found that 24 cutters, will cut all possible gears of any one pitch with sufficient practical accuracy. The range of these cutters is indicated in the following table, taken from Brown & Sharpe's "Treatise on Gearing":

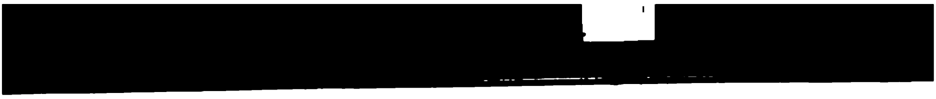
Cutter	A	cuts	12 teeth.	Cutter	M	cuts	27 to 29 teeth
"	B	"	13 "	"	N	"	30 " 33 "
"	C	"	14 "	"	O	"	34 " 37 "
"	D	"	15 "	"	P	"	38 " 42 "
"	E	"	16 "	"	Q	"	43 " 49 "
"	F	"	17 "	"	R	"	50 " 59 "
"	G	"	18 "	"	S	"	60 " 74 "
"	H	"	19 "	"	T	"	75 " 99 "
"	I	"	20 "	"	U	"	100 " 149 "
"	J	"	21 to 22 teeth	"	V	"	150 " 249 "
"	K	"	23 " 24 "	"	W	"	250 " Rack.
"	L	"	24 " 26 "	"	X	"	Rack.

These same principles of interchangeable sets of gears, with cycloidal tooth outlines, apply not only to small milled gears as above, but also as well to large cast gears with tooled or untooled tooth surfaces.

INTERCHANGEABLE INVOLUTE GEARS.

In the involute system the second condition of interchangeability is that the angle between the common tangent to the base circles, and the line of centres shall be constant.

This may be shown as follows: Draw AB, Fig. 82, a line of centres, and through P, the assumed pitch point, draw CD, and let it be the constant common tangent to all base circles from which involute tooth curves are to be drawn. Draw any pair of pitch circles tangent at P and with their centres in the line AB. About these centres draw circles tangent to CD, these would be the base circles, and CD may represent a cord that winds from one upon the other and a point in this cord will generate simultaneously involutes that will engage for the transmission of a constant velocity ratio. But this is true of *any* pair of circles that have their centres in AB, and are tangent to CD. There-



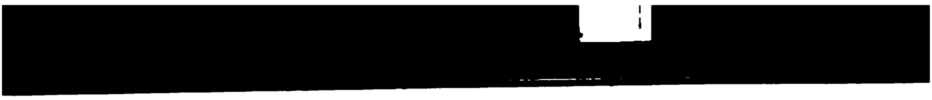
fore if the pitch is constant, any pair of gears that have the base circles tangent to the constant line CD, will mesh together properly. As in the cycloidal gears, the involute tooth curves vary with a variation in the number of teeth, and so for absolute theoretical accuracy, there would be required for each pitch, as many cutters as there might be different numbers of teeth in the gears to be cut. The variation is least at the pitch line and increases with the distance from it. The involute teeth are usually used for the finer pitches, and the cycloidal teeth for the coarser pitches and since the amount that the tooth surface extends beyond the pitch line, increases with the pitch, it will be seen that the variation in form of tooth curves is greater in the coarse pitch cycloidal gears than in the fine pitch involute gears. For this reason it is found that for each pitch, with involute gears, it is only necessary to use eight cutters, while twenty-four are used for cycloidal gears. The range is shown in the following table, which is also taken from Brown & Sharpe's "Treatise on Gearing."

No.	1	will	cut	wheels	from 135	teeth	to	racks	inclusive.
"	2	"	"	"	" 55	"	"	134	"
"	3	"	"	"	" 35	"	"	54	"
"	4	"	"	"	" 26	"	"	34	"
"	5	"	"	"	" 21	"	"	25	"
"	6	"	"	"	" 17	"	"	20	"
"	7	"	"	"	" 14	"	"	16	"
"	8	"	"	"	" 12	"	"	13	"

The curve of these cutters cannot be accurately enough constructed by laying out and finishing by hand, and so it is accomplished mechanically by machines of considerable complication, a description of which is beyond the scope of these lectures. The Pratt & Whitney method is described fully in McCord's Kinematics. It is believed that there is no published account of the Brown and Sharpe method.

LAYING OUT OF GEAR TEETH. EXACT AND APPROXIMATE METHODS.

There is ordinarily no reason why the exact cycloidal or involute curves, used for tooth outlines, should not be used



in laying out large gears or gear patterns, or the making of drawings.

Suppose that it is required to lay out a cycloidal gear, and that the pitch, and diameter of pitch, and describing circle are given. Draw the pitch circle, and from a piece of thin wood cut out a template that fits a segment of the pitch circle from the inside, as A (Fig. 83.) Cut another template that fits a segment of the pitch circle from the outside, as B. Also cut a wooden circle whose diameter equals that of the given describing circle, and fix a tracing point in its circumference. Divide the pitch circle into parts equal to the given circular pitch. Let P be one of the pitch points. Locate A so that its curved edge coincides with the pitch circle to the right of P. Roll the describing circle on A so that the epicycloid described by the tracing point shall pass through P. Next place B so that its curved edge coincides with the pitch circle at the left of P, and roll the circle on the inside of B so that the hypocycloid described by the tracing point shall pass through P. Thus the outline of one tooth is drawn, aPb. Cut an wooden template that fits accurately the tooth curve and make it fast to a wooden arm that is free to rotate about O, making the edge of the template coincide with aPb. It may now be swung successively to the other pitch points, and the tooth outline may be drawn by the template edge. This gives one side of all of the teeth. The arm may now be turned over and the other sides of the teeth may be drawn similarly.

Suppose now that it is required to lay out exact involute teeth, the pitch, pitch circle diameter, and angle of the common tangent being given. Draw the pitch circle, (Fig. 84.) and the line of centres, AB. Through the pitch point P, draw CD, the common tangent making the given angle β with the line of centres. Draw the base circle about O, tangent to CD. Cut a wooden template that fits the base circle from the inside, as EF; wind on this template a fine cord carrying a pencil at its end, and then unwind this allowing the pencil to trace an involute curve which will be a correct tooth form. Let a template, cut to fit this involute, be attached to an arm free to rotate about O, and the tooth outlines may be drawn as before.

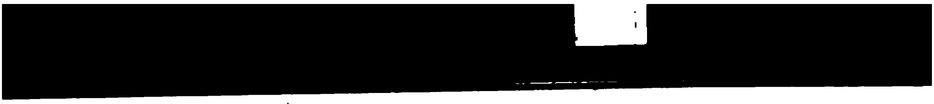
Sometimes, however, it is desirable to have the means of getting approximate tooth outlines by some quicker method than the above, and the Willis Odontograph may be used. Let AB and CD, (Fig. 85,) be any pair of pitch circles corresponding to a certain velocity ratio. Draw EF through the pitch point P making any angle θ with the line of centres; Select any two points in this line as G and H, and with these as centres, draw arcs of circles tangent to each other at some other point of EF, as K. Evidently the normal to these curves at the point of contact passes through the virtual centre of the pitch circles AB and CD, and therefore, for the instant, these circles would serve for tooth profiles to transmit the required velocity ratio. Let a circle now be drawn whose centre is in line of centres, and whose circumference passes through K and P. This circle may be a describing circle for cycloidal teeth, and K may be the describing point, and if the circles turn, constantly tangent at P without slipping, K will describe epi- and hypocycloids that would satisfy the condition of constant velocity ratio. If now the mean radii of curvature be determined and the centres of curvature of these curves thus located on the line EF, normal to the curve at K, arcs of circles may be drawn from them as centres, that would be approximations to the cycloidal curves, and which would, if the teeth were not too long, serve for tooth profiles to transmit an approximately constant velocity ratio. As long as the angle θ, and the distance PK are constant, the approximation is to curves generated by a constant describing circle; because, since the centre of the describing circle must be on the line of centres, and since its circumference must pass through P and K, therefore if θ and PK are constant, the describing circle cannot vary in size.

A consideration of these principles led Prof, Willis to construct his Odontograph. This instrument consists of a piece of card board or sheet metal, (Fig. 86.) The edge from O to A and from O to D, is graduated in a scale of equal parts. θ is made $= 75°$ so that, when one edge of the odontograph is made to coincide with a radius of the pitch circle passing through the pitch point P of a tooth,

(Fig. 87), the line EF is the position of the normal to the middle or the epicycloid ab, and the hypocycloid cd, whose pitch points b and c are at a distance from P = half the pitch. A table is supplied that gives the values of the mean radii of curative of ab and cd, in terms of the scales OA and OB, for any pitch, and so the centres may be located and the approximate arcs drawn.

To illustrate the method of use of the Willis odontograph see (Fig. 87.) GH is the pitch circle of a gear, and b, P and c are pitch points. Make the point O of the odontograph coincide with P, and the edge OD with the radical line through P. Find the number in the table under "Centres for Flanks of Teeth" corresponding to the given circular pitch and number of teeth. Locate this number on the scale OA; this is the centre of the circular arc which, drawn through C, approximates the hypocycloid cd. Next find the number in the table under "Centres for Faces of Teeth" corresponding to the given circular pitch and number of teeth. Locate this on the scale OB and this is the centre of the arc ba, which approximates the epicycloid ab through b. Repetition of the process gives the outlines of all of the teeth. After one pair of centres is found by the odontograph, of course, circles through these centres about the gear centre, will be the loci of all centres and they may be located without the odontograph. For the drawing of approximate single curve or involute teeth, the Willis odontograph takes the form shown in (Fig. 88.) O is located at a pitch point in the given pitch circle, and OB is made to coincide with the radius of the pitch circle. OA is so graduated, that if a number be read off from O equal to the radius of the pitch circle in inches, the centre of the approximate involute curve through O will be located.

Another and entirely different Odontograph is the one invented by Prof. S. W. Robinson, and called the Templet Odontograph, and shown in Fig. 89. The edge AB is a certain logarithmic spiral, and AC is its evolute. AB is graduated in a scale of equal parts. Let GH be an arc of the given pitch circle, and D the centre point of a tooth. Lay off DL = half the tooth thickness. A value is now



found in a table that is dependent on the diameters of the two wheels that are to work together, and the number of teeth in the wheel whose teeth are to be drawn. This number is located on the scale AB, and this point is made to coincide with the point L, and also the curve AC is made tangent to the tangent EF to the pitch circle at D. The odontograph is now properly located and the edge LB may be used as a templet to draw the face of the tooth. The flanks in this system are radial and so may be easily drawn. By the use of different tabular numbers, involute approximate teeth may be drawn. This odontograph may be conveniently fastened in proper position to an arm that is free to rotate about the centre of the gear, and swung successively so that the curved edge coincides with the several pitch points and half of the faces may be drawn; it may then be turned over and the rest may be drawn.

To assist in the practical proportioning of gear wheels the following notation and tables are taken from Brown and Sharpe's "Treatise on Gearing":

D = diameter of the addendum circle = outside diameter of the gear.

D' = diameter of the pitch circle.

D'' = the working depth of the teeth.

P = diametral pitch = number of teeth per inch of pitch circle diameter.

P' = circular pitch = space on the pitch circle occupied by a tooth and a space.

N = number of teeth.

t = thickness of the tooth at the pitch line.

s = addendum = radial distance from pitch circle to point of tooth, therefore $2s = D''$.

f = clearance = depth of space in excess of the working depth.

$s + f$ = depth of space below the pitch line.

$D'' + f$ = whole depth of space.

[redacted]

P′	P	t	s	D″	s + f	D″ + f
2	1.5708	1.0000	.6366	1.2732	.7366	1.3732
1 7/8	1.6755	.9375	.5968	1.1937	.6906	1.2874
1 3/4	1.7952	.8750	.5570	1.1141	.6445	1.2016
1 5/8	1.9333	.8125	.5173	1.0345	.5985	1.1158
1 1/2	2.0944	.7500	.4775	.9549	.5525	1.0299
1 7/16	2.1855	.7187	.4576	.9151	.5294	.9870
1 3/8	2.2848	.6875	.4377	.8754	.5064	.9441
1 5/16	2.3936	.6562	.4178	.8356	.4834	.9012
1 1/4	2.5133	.6250	.3979	.7958	.4604	.8583
1 3/16	2.6156	.5937	.3780	.7560	.4374	.8156
1 1/8	2.7925	.5625	.3581	.7162	.4143	.7724
1 1/16	2.9568	.5312	.3382	.6764	.3913	.7295
1	3.1416	.5000	.3183	.6366	.3683	.6866
15/16	3.3510	.4687	.2984	.5968	.3453	.6437
7/8	3.5904	.4375	.2785	.5570	.3223	.6007
13/16	3.8666	,4062	.2586	.5173	.2993	.5579
3/4	4.1888	.3750	.2387	.4775	.2762	.5150
11/16	4.5696	.3437	.2189	.4377	.2532	.4720
2/3	4.7124	.3333	.2122	.4244	.2455	.4577
5/8	5.0265	.3125	.1989	.3979	.2301	.4291
9/16	5.5851	.2812	.1790	.3581	.2071	.3862
1/2	6.2832	.2500	.1592	.3183	.1842	.3433
7/16	7.1808	.2187	.1393	.2785	.1611	.3003
2/5	7.8540	.2000	.1273	.2546	.1473	.2746
3/8	8.3776	.1875	.1194	.2387	.1381	.2575
1/3	9.4248	.1666	.1061	.2122	.1228	.2289
5/16	10.0531	.1562	.0995	.1989	.1151	.2146
2/7	10.9956	.1429	.0909	.1819	.1052	.1962
1/4	12.5664	.1250	.0796	.1591	.0921	.1716
2/9	14.1372	.1111	.0707	.1415	.0818	.1526
1/5	15.7080	.1000	.0637	.1273	.0737	.1373
3/16	16.7552	.0937	.0597	.1194	.0690	.1287
1/6	18.8496	.0833	.0531	.1061	.0614	.1144
1/7	21.9911	.0714	.0455	.0910	.0526	.0981
1/8	25.1327	.0625	.0398	.0796	.0460	.0858
1/9	28.2743	.0555	.0354	.0707	.0409	.0763
1/10	31.4159	.0500	.0318	.0637	.0368	.0687
1/16	50.2655	.0312	.0199	.0398	.0230	.0429

P	P′	t	s	D″	$s+f$	D″$+f$
½	6.2832	3.1416	2.0000	4.0000	2.3142	4.3142
¾	4.1888	2.0944	1.3333	2.6666	1.5428	2.8761
1	3.1416	1.5708	1.0000	2.0000	1.1571	2.1571
1¼	2.5133	1.2566	.8000	1.6000	.9257	1.7257
1½	2.0944	1.0472	.6666	1.3333	.7714	1.4381
1¾	1.7952	.8976	.5714	1.1429	.6612	1.2326
2	1.5708	.7854	.5000	1.0000	.5785	1.0785
2¼	1.3963	.6981	.4444	.8888	.5143	.9587
2½	1.2566	.6283	.4000	.8000	.4628	.8628
2¾	1.1424	.5712	.3636	.7273	.4208	.7844
3	1.0472	.5236	.3333	.6666	.3857	.7190
3½	.8976	.4488	.2857	.5714	.3306	.6163
4	.7854	.3927	.2500	.5000	.2893	.5393
5	.6283	.3142	.2000	.4000	.2314	.4314
6	.5236	.2618	.1666	.3333	.1928	.3595
7	.4488	.2244	.1429	.2857	.1653	.3081
8	.3927	.1963	.1250	.2500	.1446	.2696
9	.3491	.1745	.1111	.2222	.1286	.2397
10	.3142	.1571	.1000	.2000	.1157	.2157
11	.2856	.1428	.0909	.1818	.1052	.1961
12	.2618	.1309	.0833	.1666	.0964	.1798
13	.2417	.1208	.0769	.1538	.0890	.1659
14	.2244	.1122	.0714	.1429	.0826	.1541
15	.2094	.1047	.0666	.1333	.0771	.1438
16	.1963	.0982	.0625	.1250	.0723	.1348
17	.1848	.0924	.0588	.1176	.0681	.1269
18	.1745	.0873	.0555	.1111	.0643	.1198
19	.1653	.0827	.0526	.1053	.0609	.1135
20	.1571	.0785	.0500	.1000	.0579	.1079
22	.1428	.0714	.0455	.0909	.0526	.0980
24	.1309	.0654	.0417	.0833	.0482	.0898
26	.1208	.0604	.0385	.0769	.0445	.0829
28	.1122	.0561	.0357	.0714	.0413	.0770
30	.1047	.0524	.0333	.0666	.0386	.0719
32	.0982	.0491	.0312	.0625	.0362	.0674
34	.0924	.0462	.0294	.0588	.0340	.0634
36	.0873	.0436	.0278	.0555	.0321	.0599
38	.0827	.0413	.0263	.0526	.0304	.0568
40	.0785	.0393	.0250	.0500	.0289	.0539
42	.0748	.0374	.0238	.0476	.0275	.0514
44	.0714	.0357	.0227	.0455	.0263	.0490
46	.0683	.0341	.0217	.0435	.0252	.0469
48	.0654	.0327	.0208	.0417	.0241	.0449
50	.0628	.0314	.0200	.0400	.0231	.0431
56	.0561	.0280	.0178	.0357	.0207	.0385
60	.0524	.0262	.0166	.0333	.0193	.0360

NON-CIRCULAR WHEELS.

Only circular centrodes or pitch curves, correspond to a constant velocity ratio; and by making the pitch curves of proper form, almost any variation in the velocity ratio may be produced. Thus a gear whose pitch curve is an ellipse, rotating about one of its foci, may engage with another elliptical gear, and if the driver have a constant angular velocity, it will communicate to the follower a constantly varying velocity; and if the follower be rigidly attached to the crank of a slider crank chain, the slider will have a quick return motion. This is sometimes applied to shaping and slotting machines. When more than one fluctuation of velocity per revolution is required, it may be obtained by means of "lobed gears;" *i. e.*, gears in which the curvature of the pitch curve is several times reversed. It will be clear that, if a describing circle be rolled on these non-circular pitch curves, the tooth outlines will vary in different parts, and so, in order to cut such gears, many cutters would be required for each gear. Practically this would be too expensive; and so, when such gears are required, the pattern is accurately made, and the cast gears are used without "tooling" the tooth surfaces.

BEVEL GEARS.

All transverse sections of spur gears are the same, and their axes intersect at infinity. Spur gears serve to transmit motion between parellel shafts. It is necessary also to transmit motion between shafts whose axes intersect; in this case the pitch cylinders become pitch cones, the teeth are built upon these conical surfaces, and the resulting gears are called bevel gears.

To illustrate, let a and b. (Fig. 90,) be the axes between which the motion is required to be transmitted, with a velocity ratio, given by the ratio of the line A to the line B. Draw a line CD, parallel to a, and at a distance from it equal to the line A. Also draw the line CE, parallel to b, and at a distance from it equal to the line B. Join the point of intersection of these lines to the point C, the intersection of the axes given. This gives the line CF, which is

contact of two pitch cones that will roll together to transmit the required velocity ratio. For, mc÷nc=A÷B and if it be supposed that there are cones so thin that they may be considered cylinders, their radii being equal to mc and nc, it will be clear that they would roll together, if slipping be prevented, to transmit the required velocity ratio. But all pairs of radii of these pitch cones have the same ratio=mc÷nc, and therefore any pair of frusta of the pitch cones may be used, and they will roll together to transmit the required velocity ratio.

Of course, in order to insure this result, slipping must be prevented, and to accomplish this, teeth are built upon the selected frusta of the pitch cones, which correspond to the teeth in spur gears. The theoretical determination of these teeth may be explained as follows: 1st, cycloidal teeth. If a cone A, (Fig. 91,) be rolled upon another cone B, an element bc of the cone A will generate a conical surface and a spherical section of this surface adb is called a spherical epicycloid. Also if a cone A, (Fig. 92,) roll on the inside of another cone C, an element bc of A, will generate a conical surface, a spherical section of which bda, is called a spherical hypocycloid. If now the three cones B and C and A, be supposed to roll together so that they are always tangent to each other on one line, as the cylinders were in the case of spur gears, there will be two conical surfaces generated by an element of A, one upon the cone B and another upon the cone C, and these may be used for tooth surfaces that will serve to transmit the required constant velocity ratio. Because, since the line of contact of the cones is the instantaneous axis of the relative motion of the cones, it follows that a plane normal to the motion of the describing element of the generating cone at any time, will pass through this instantaneous axis. And also, since the describing element is always the line of contact between the generated tooth surfaces, therefore the normal plane to the line of contact of the tooth surfaces always passes through the virtual axis, and the condition of rotation with a constant velocity ratio is fulfilled.

2d. Involute Teeth. If two pitch cones be in contact along an element, a plane may be passed through this ele-

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ment, making an angle (say 75°) with the plane of the axes of the cones. Tangent to this plane there may be two cones, whose axes coincide with the axes of the pitch cones. If now a plane be supposed to wind off from one base cone and on to the other, the line of tangency of the plane with one cone, will leave the cone and advance in the plane toward the other cone, and will generate simultaneously upon the pitch cones, cylindrical surfaces, and spherical sections of these surfaces will be spherical involutes. These surfaces may be used for tooth surfaces, and will transmit the required constant velocity ratio, because the tangent plane is the constant normal to the tooth surfaces, at their line of contact, and this plane passes through the virtual axis of the pitch cones.

In order to determine the tooth outlines absolutely it would be necessary to draw the required curves on a spherical surface and then join all points of these curves to the point of intersection of the axes of the pitch cones. Practically this would be impossible, and so an approximate method is used.

If the frusta of pitch cones be given, B and C, (Fig. 93,) then points in the base circles of the cones, as L, M and K, will move always in the surface of a sphere which may be represented by the circle LAKM. Properly the tooth curves should be laid out on the surface of this sphere, and joined to the centre of the sphere to generate the tooth surfaces. Draw cones LGM and NHK tangent to the sphere on circles represented in projection by the lines LM and MK. If now tooth curves be drawn on these cones, with the base circle of the cones as pitch circles, they will very closely approximate the tooth curves that should be drawn on the spherical surface. But a cone may be cut along one of its elements, and rolled out, or developed upon a plane. Let MGN be a section of the cone LGN developed, and let MHD be a section of the cone MHK developed. The circular arcs MD and MN may be used just as pitch circles are in the case of spur gears, and the teeth may be laid out in exactly the same way, the curves being either cycloidal or involute, as is required. Then the developed cones may be wrapped back and the curves drawn may serve as directrices for the tooth surfaces, all of which converge to the centre of the sphere of motion.

The teeth of spur gears may be cut by means of milling cutters, because all transverse sections are alike ; but with bevel gears the conditions are different. The tooth surfaces are conical surfaces, and therefore the curvature varies continually from one end of the tooth to the other. Also the thickness of the tooth, and the width of the space vary continually from one end to the other. But the curvature of a milling cutter, and its thickness cannot vary, and therefore a milling cutter cannot cut an accurate bevel gear. Small bevel gears are however, cut with milling cutters, that are near enough correct for practical purposes. The cutter is made as thick as the narrowest part of the space between the teeth, and its curvature is made that of the middle of the tooth. Two cuts are made for each space. Let (Fig. 94) represent a section of the cutter. For the first cut it is set, relatively to the gear blank, so that the pitch point a of the cutter travels toward the apex of the pitch cone, and for the second cut, so that the pitch point b travels toward the apex of the pitch cone. It will be seen that this method gives an approximation to the required form. Gears cut in this manner usually need to be filed slightly before they work satisfactorily.

Bevel gears with absolutely correct tooth surfaces may be made by planing. Suppose a planer in which the tool point travels always in some line through the apex of the pitch cone. Then suppose that as it is slowly fed down the tooth surface it is guided along the required tooth curve by means of a templet. From what has preceded it will be clear that the tooth so formed will be correct. Planers embodying these principles have been designed and constructed by Mr. Corliss of Providence, and Mr Gleason of Rochester, with the most satisfactory results.

Spur Gears serve to communicate motion between axes that are parallel, and Bevel Gears between axes that intersect ; but it is sometimes necessary to communicate motion between axes that are neither parallel nor intersecting. If the parallel axes be turned out of parallelism, or if intersecting axes be moved into different planes so that they no longer intersect, the pitch surfaces become hyperboloic peraboloids, in contact with each other along a straight line which is the

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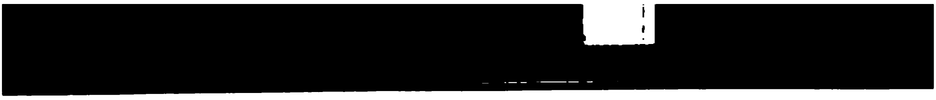
generatrix of the pitch surfaces. These hyperbolic peraboloids may be rolled upon each other, circumferential slipping being prevented, and will transmit a constant velocity ratio. There is, however, necessarily a slipping of the elements of the surfaces upon each other, parallel to themselves. Teeth may be built on these pitch surfaces, and they may be used for the transmission of motion between shafts that are not parallel, nor in the same plane. Such gears are called "Skew Bevel Gears." The difficulties of construction and the additional friction due to the slipping along the elements, make them very undesirable in practice, and also there is very seldom a place where they cannot be replaced by some other form of connection.

A very complete discussion of the subject of Skew Bevel Gears may be found in Prof. MacCord's "Kinemetics."

SPIRAL GEARING.

If line contact between the teeth of the gears is not considered an essential, there is a much wider range of choice of gears to connect shafts that are neither parallel nor intersecting. Suppose that A and B, (Fig. 95,) are axes of rotation in different planes, both planes being parallel to the plane of the paper. Let EF and GH be cylinders on these axes. Any line may now be drawn through S, either between A and B, or coinciding with either one of them, and this line, say DS, may be taken as the common tangent to helical, or screw lines, drawn on the cylinders EF and GH. If helical or screw teeth be built on both cylinders, DS being the common tangent to their surfaces at S, we shall have what is called "Spiral Gears." Each one is a portion of a many threaded screw. The contact in these gears is point contact; in practice, the point of contact becomes a very limited area. When the angle between the shafts is made equal to 90° and one gear has only one, two or three threads, we have a special case of Spiral gearing that is known as Worm, or Screw gearing. In this special case the gear with a single or double thread, and is called the "worm," while the other gear, which is still a *many* threaded screw, is called the "worm wheel." If the worm has a single thread it is really a one



toothed gear, because while it makes one revolution, it causes one tooth of the worm wheel to pass the line of centres; therefore, in order that the worm wheel shall make one revolution, the worm must make as many revolutions as there are teeth in the worm wheel. From this it will be seen that the velocity ratio is equal to $1 \div$ number of teeth in the worm wheel. If the worm has a double thread, then two teeth of the worm wheel pass the line of centres per revolution of the worm, and the velocity ratio becomes equal to $2 \div$ number of teeth in the worm wheel. From this it will be seen that these gears are particularly well adapted to the cases where a very great change of angular velocity is required with only one pair of gears.

If a section of a worm and worm wheel be made on a plane passing through the axis of the worm, and normal to the axis of the worm wheel, the form of the teeth will be the same as that of a rack and pinion; in fact, the worm, if moved parallel to its axis, would transmit rotary motion to the worm wheel. From our consideration of racks and pinions it will be clear that if the involute system be used, the sides of the worm teeth, or rather the worm thread, will be straight lines. this simplifies the cutting of the worms in the lathe, because it admts of the use of a tool that may be sharpened by grinding by any workman. If the worm wheel were only a thin plate, the teeth would need simply to be formed like those of an ordinary spur gear. But since it must have some thickness, and since all other sections parallel to that through the axis of the worm, as CD and AB, (Fig. 96,) show a different form of tooth, it is necessary to make the teeth of the worm wheel of different form from those of a spur gear, if there is to be contact between the worm and worm wheel anywhere except in the plane EF, (Fig. 96.) This would seem to involve great difficulties of construction, but they are overcome in practice as follows. A duplicate of the worm is made of tool steel, and flutes are cut in it parallel to the axis, so as to make it into a cutter, and it is then tempered. It is then mounted in a frame-work in the same relation to the worm wheel that the worm is to be when they are finished, and in position for working, except that the distance between centres

is somewhat greater, and is capable of being gradually reduced. Both are then rotated with the required velocity ratio by means of gearing properly arranged, and the cutter or "hob" is fed against the worm wheel till the distance between centres is made the required value. The teeth of the worm wheel are "roughed out" before they are "hobbed" to finish. By this means the worm is made to cut its own worm wheel, and therefore the tooth forms are correct. This subject is more fully treated in "Unwin's Elements of Machine Design" and in "Brown and Sharpe's Treatise on Gearing."

COMPOUND SPUR GEAR CHAINS.

Spur gear chains, like chains connected by turning and sliding pairs, may be compound ; *i. e.*, they may contain links that carry more than two elements. Thus in Fig. 97, the link a carries three elements, as does also the link d. In the latter case the teeth of d must be counted as two elements, as by means of them d is paired with both b and c. In the case of the three link spur gear chain, the wheels b and c meshed with each other, and a point in the pitch circle of c moved with the same linear velocity as a point in the pitch circle of b, but in the opposite sense. In Fig. 97, points in all of the pitch circles have the same linear velocity, since they roll on each other without slipping, but c and b now rotate in the same sense. Hence, it is seen that the introduction of the wheel d has simply reversed the sense of rotation, without changing the velocity ratio. The size of the wheel d, which is called an "idler," has no effect upon the motion of c and b since it simply receives, upon its pitch circle, a certain linear velocity from c, and transmits it unchanged to b. From this it will be also clear, that the insertion of any number of idlers does not affect the velocity ratio of c to b, but that each added idler reverses the sense of the motion. Thus if the number of idlers be odd, c and b will rotate in the same sense : and if the number of idlers be even, c and b will rotate in the opposite sense.

It has already been shown that if parallel lines be drawn through the centres of rotation of a pair of gears, and dis-

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tances be laid off on these lines from the centres of rotation, that are inversely proportional to the angular velocities of the gears, then a line joining the points so determined will cut the line of centres of the gears in a point which is the virtual centre of the gears. In Fig. 97, since the rotation is in the same sense, the lines have to be laid off on the same side of the line of centres. But also it is known that the pitch radii are inversely proportional to the angular velocities of the gears, and so it is only necessary to draw a tangent to the pitch circles of c and b, and the intersection of this line, with the line of centres, is the virtual centre of c and b, Obc. The centrodes of c and b are circles through the point Obc, about the centres of c and b, as c_1 and b_1. It will now be clear that this four link mechanism may be replaced by a three link mechanism, in which c, is an annular wheel and b, a pinion. The four link mechanism, however, is usually more convenient in practice, because it occupies so much less space,

The other principal form of spur gear chain is shown in Fig. 98. In this case the wheel d has two sets of teeth, of different pitch diameter, one pairing with c, and the other with b. The point Obd does not now have the same linear velocity as Ocd, but a velocity greater or less in the proportion to the ratio of the radii of those points, as points in the gear d. The angular velocity ratio may be obtained as follows:

$$\frac{\text{angular veloc. c}}{\text{angular veloc. d}} = \frac{DOcd}{COcd} \quad \text{also} \quad \frac{\text{angular veloc. d}}{\text{angular veloc. b}} = \frac{BObd}{DObd}$$

Multiplying,

$$\frac{\text{angular veloc. c}}{\text{angular veloc. b}} = \frac{DOcd \times BObd}{COcd \times DObd} \quad \text{or} \quad = \frac{DOcd}{DObd} \times \frac{BObd}{COcd}$$

Inspecting the last term it is seen that the numerator consists of the product of the radii of the "followers," and the denominator consists of the product of the radii of the "drivers."

Of course the diameters or numbers of teeth could just as well be substituted for the radii. And it may be generally stated, that the angular velocity of the first driver, is to the angular velocity of the last follower, as the product of the number of teeth of the followers, is to the product of the num-

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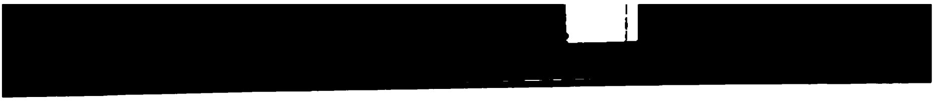
ber of teeth of the drivers. This applies equally well to compound spur gear trains that have more than three axes. Therefore in any spur gear chain the velocity ratio = product of number of teeth in the followers, divided by the product of number of teeth in the drivers, and the sense of rotation is reversed if the number of intermediate axes is even, and is not reversed if the number is odd.

CAMS.

A non-circular cylinder or disc, that is used to communicate motion to some link of a kinematic chain, by line contact, is called a cam.

In the design of cams it is customary to consider a number of definite positions that the follower must occupy, when the driver occupies certain other positions, and then from these data to derive the cam curve.

Case I. The follower is guided in a straight line, and the contact of the driving cam with the follower is always in this line. The line may be in any position relatively to the centre of rotation of the cam, and therefore it is a general case. (See Fig. 99.) MN is the line in which the point of the follower that bears on the cam is constrained to move, and O is the centre of rotation of the cam. About O as a centre, draw a circle tangent to MN at J. Then A, B, C, etc., are definite points in the cam. When the point A is at J, the follower point must be at A′; when B is at J, the follower point must be at B′, and so on through an entire revolution. Through A, B, C, etc., draw lines tangent to the circle. With O as a centre, and OA′ as a radius, draw a circular arc A′ A″, intersecting the tangent through A, at A″. Then A″ will be a point in the cam curve. For, if A be rotated to J, AA″ will coincide with JA′, and A″ will coincide with A′, and the cam will hold the follower in the required position. The same process for the other positions given locates other points of the cam curve, and if a smooth curve be drawn through these points, the cam will be determined. Very often, in order to reduce the amount of friction, a roller is attached to the follower, and rests on the cam, the motion being communicated through it. When this is the case it

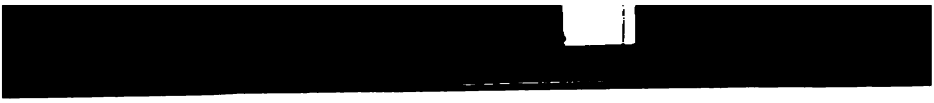


will be seen that the curve found as above, will be the path of the axis of the roller, and the real cam curve will then be a curve drawn inside of, and parallel to the path of the axis of the roller, at a distance from it equal to the radius of the roller.

In this case the line of contact between the cam and the follower was always in the line of motion of the follower; but this condition is not always met in practice. Often a cam engages with a surface instead of with a roller or a point, and it may then be impossible for the line of contact to be always in the same line.

Case II. The cam engages with a surface of the follower, and this surface is guided so that it moves parallel to itself. This method is due to Professor J. H. Barr. (See Fig. 100.) O is the centre of the rotation of the cam, and 1, 2, 3, etc., are positions of the follower surface that are occupied successively, when the lines of the cam A, B, C, etc., coincides with the vertical line through C. It is required to draw a cam curve that shall constrain the motion as required. Produce the vertical line through O, cutting the positions of the follower surface in A′, B′, C′, etc. With O as a centre and radii OB′, OC′. etc. draw arcs cutting the lines B, C, D, etc., in the points B″, C″, D″, etc. Position 1 is the lowest position of the follower surface, therefore A must be in contact with the follower surface in the vertical line through O; because if the tangency be at any other point, the motion in one direction or the other will lower the follower, which is not allowable. A is therefore one point in the cam curve. If now a line be drawn through B″ at right angles to B″O, and B″O be rotated till it coincides with B′O, the line just drawn will coincide with the position of the follower surface 2B′. But the cam curve must be tangent to this line when B coincides with B′O, and therefore the line just drawn is a line to which the cam curve must be tangent. Similar lines may be drawn through the points C″, D″, etc., and each of them will be a line to which the cam curve must be tangent. Therefore if a smooth curve be drawn, tangent to all of these lines, it will be the required cam curve.

Case III. This is the same as Case II, except that the



follower surface instead of moving parallel to itself vibrates about some axis as O′, Fig. 101. The solution is the same as in Fig. 100, except that instead of drawing the lines through B″, C″, etc., perpendicular to B, C, etc., they are drawn so that they make the same augle with B, C, etc., that the follower surface makes with the vertical through O, when B, C, etc., coincide with that vertical.

In the cases so far considered, the cam has driven the follower in only one direction ; gravity, a spring or some other force must hold it in contact with the cam. When it is necessary that the cam should drive the follower in both directions, the cam surface must be double, *i. e.*, it takes the form of a groove in which works a pin or roller attached to the follower. The same principles for laying out the curves apply as before.

KINEMATICS OF BELTING.

In Fig. 102, let A and B be two cylindrical surfaces, free to rotate about their axes, and let CD be a common tangent to them. Suppose that CD represents an inextensible connection between the two cylinders. Since it is inextensible, D and C must have the same linear velocity, and therefore their angular velocities are inversely proportional to their distances from their centres of rotation ; or the angular velocity of D ÷ the angular velocity of C = Cb ÷ Da ; but Cb and Da are the radii of the cylinders, and therefore the ratio of angular velocities of the cylinders connected by an inextensible connecter, is the inverse ratio of the radii of the cylinders. Suppose now that the cylinders become pulleys and the tangent line is a belt. Let C′ D′ be drawn ; this becomes a part of the belt, making it endless, and rotation may now be continuous, the belt will remain always tangent to the pulleys, and will transmit rotation such that the angular velocity ratio will constantly be the inverse ratio of the radii of the pulleys.

The case considered corresponds to a crossed belt ; but the same reasoning applies equally well to an open belt. (See Fig. 103.) A and B are two pulleys, and CDD′C′C is an open belt, and since the points C and D are connected by a belt that is practically inextensible, the linear velocity of C

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and D is the same, and therefore their angular velocities are to each other inversely as their radii.

If the pulleys in either case were pitch cylinders of gears, the conditions of velocity ratio would be the same. In the first case, however, the sense of motion is reversed, and in the second case it is not; therefore the first corresponds to gears that mesh directly with each other, while the second corresponds to the case in which the gears are connected by an idler, or to the case of annular gear and pinion. Of course it is necessary that a belt should have some thickness; and, since the centre of pull is the centre of the belt, it is necessary to add to the radius of the pulley, half of the thickness of the belt. The motion, however, that is communicated by means of belting does not need to be absolutely correct, and therefore, in practice it is usually customary to neglect the thickness of the belt. The proportioning of pulleys for the transmission of any required velocity ratio will now be a very simple matter. Let a practical case be taken for illustration. A line shaft runs 150 revolutions per minute, and is supported by hangers with 16″ "drop." It is required to transmit motion from this shaft to a dynamo that must run 1800 revolutions per minute. A 30″ pulley is the largest one that can be conveniently used with 16″ hangers. Let x = the diameter of the required pulley for the dynamo, then from what has preceded, $x \div 30 = 150 \div 1800$, and therefore $x = 2.5''$. But the minimum size of pulley that can be used on a dynamo, varies with the capacity from 4 to 10″; suppose, in this case, that it is $= 6''$. It is then impossible to obtain the required velocity ratio with one change of speed, *i. e.*, with one belt. Therefore two changes of speed must be obtained by the introduction of a counter shaft. By this means the velocity ratio is divided into two factors. If it is wished to have the same change of speed from the line shaft to the counter, as from the counter to the dynamo, then each velocity ratio would be $\sqrt{180 \div 150} = \sqrt{12} = 3.46$. But this gives an inconvenient fraction, and there is no need that the two ratios should be equal. Let the factors be 3 and 4. (See Fig. 104.) A represents the line shaft, B the counter, and C the dynamo shaft. The pulley on the

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line shaft is 30″ and the speed is to be three times as great at the counter, and therefore the pulley must have a diameter one-third as great = 10″. Also the pulley on the dynamo is 6″ diameter, and the counter shaft is to run one-fourth as fast, and therefore the pulley on the counter opposite the dynamo pulley, must be four times as large as the dynamo pulley = 24″.

A belt may be shifted from one part of a pulley to another, by means of side pressure against the side which advances toward the pulley. Thus if, in Fig. 105, the rotation be as indicated by the arrow, and side pressure be applied at A as shown, the belt will be pushed to one side as is shown by the dotted lines, and will consequently be carried into some new position on the pulley further to the left, as it advances. From this it will be seen, that in order that a belt may maintain its position on a pulley, the centre line of the advancing side of the belt must be in a plane perpendicular to the axis of the pulley. If this condition be fulfilled the belt will run and transmit the required motion, regardless of the position of the shafts relatively to each other. In Fig. 106, the axes AB and CD are parallel to each other, and the above stated condition is fulfilled, and the belt will run correctly; but if the axis CD be turned into some new position as C′D′, then the side of the belt that advances toward the pulley E cannot have its centre line in a plane perpendicular to the axis and therefore it will run off. But if a plane be passed through the line CD, perpendicular to the plane of the paper, then the axis may be swung in this plane in such a way that the necessary condition shall be fulfilled, and the belt run properly. This gives what is known as a "twist" belt, and when the angle between the shafts becomes = 90°, it is a "quarter twist" belt. To make this clearer, see Fig. 107. Rotation is transmitted from A to B by an open belt, and it is required to turn the axis of B out of parallelism with that of A. The sense of rotation is as indicated by the arrows. Draw the line CD. If now the line CD be supposed to pass through the centre of the belt at C and D, it may become an axis, and the pulley B and the part of the belt FC, may be turned about it, while the pulley A and the part of the belt

ED remain stationary. It will be seen that during this motion the centre line of the part of the belt CE, which is the part that advances toward the pulley B when rotation occurs, is always in a plane perpendicular to the axis of the pulley B. The part ED, since it has not been moved, has also its centre line in a plane perpendicular to the axis of A. Therefore the pulley B may be swung into any angular position about CD as an axis, and the condition of proper belt transmission will not be interfered with.

If the axes intersect, the motion can be transmitted between them by belting only by the use of "guide," or "idler" pulleys. Let AB and CD, Fig. 108, be intersecting axes, and let it be required to transmit motion from one to the other by means of a belt running on the pulleys E and F. Draw centre lines EK and FH through the pulleys. Draw the circle G, of any convenient size, tangent to the lines EK and FH. In the axis of the circle G let a shaft be placed, on which are two pulleys, their diameters being equal to that of the circle G. These will serve as guide pulleys for the upper and lower sides of the belt, and by means of them, the centre lines of the advancing parts, of both sides of the belt, will be kept in planes perpendicular to the axis of the pulley toward which they are advancing, and so the belts will run properly, and the motion will be transmitted as required.

An analogy will be noticed between gearing and belting for the transmission of rotary motion. Spur gearing corresponds to an open or crossed belt, transmitting motion between parallel shafts. Bevel gears correspond to a belt running on guide pulleys, transmitting motion between intersecting shafts. Skew bevel, and Spiral gearing correspond to a "twist" belt, transmitting motion between shafts that are neither parallel nor intersecting.

If a flat belt be put on a pulley that is turned "crowning," as in Fig. 109, the tension on AB will be greater than on CD, and the belt will tend to be twisted into the position shown by the dotted lines EF, and as rotation goes on, the belt will be carried toward the high part of the pulley, or will tend to run in the middle of the pulley. This is the

reason why nearly all belt pulleys, except those on which the belt has to be shifted into different positions, are turned "crowning."

CONE PULLEYS.

In performing different operations on a machine, or the same operations on materials of different degrees of hardness, different speeds are required. The simplest way of obtaining them is by the use of cone pulleys. One pulley has a series of steps, and the opposing pulley has a corresponding series of steps. By shifting the belt from one pair of steps to the other, the velocity ratio is of course changed. It is clear, since the same belt is used on all the pairs of steps, that they must be so proportioned that the belt length for all the pairs shall be the same; otherwise the belt would be too tight on some of the steps, and too loose on others. Let the case of a crossed belt be first considered. The length of a crossed belt may be expressed by the following formula:

Let L = the length of the belt.
" d = the distance between the centres of rotation.
" R = the radius of the larger pulley.
" r = " " " " smaller " .

See Fig. 110.

Then $L = 2\sqrt{d^2 - (R+r)^2} + (R+r)(\pi + 2$ arc whose sine is $R + r \div d.)$

In the case of a crossed belt, if the size of the steps be changed so that the sum of their radii remains constant, the belt length will be constant. For in the formula the only variables are R and r, and these terms only appear in the formula as $R+r$; but $R+r$ is by hypothesis constant; therefore any change that is made in the values of R and r, so long as their sum is constant, will not affect the value of the equation, and hence the belt length will be constant.

It will now be easy to design cone pulleys for crossed belt. Suppose given a pair of steps that transmit a certain velocity ratio, and it is required to find a pair of steps that will transmit some other velocity ratio, the length of belt being the same in both cases.

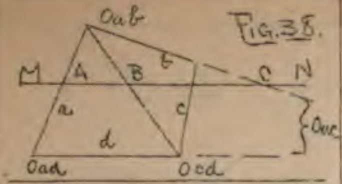

Fig. 38.

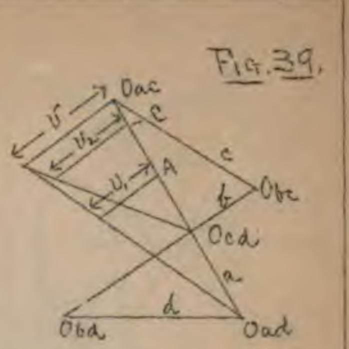

Fig. 39.

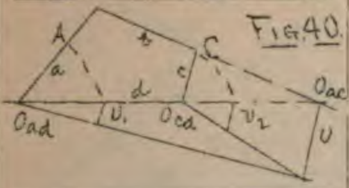

Fig. 40.

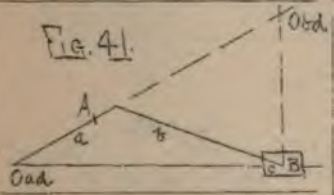

Fig. 41.

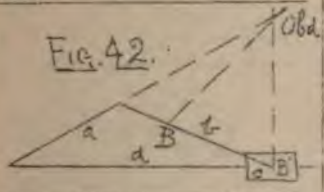

Fig. 42.

Fig. 43.

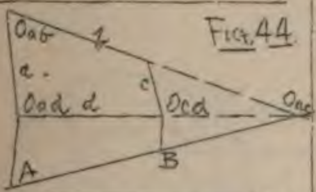

Fig. 44.

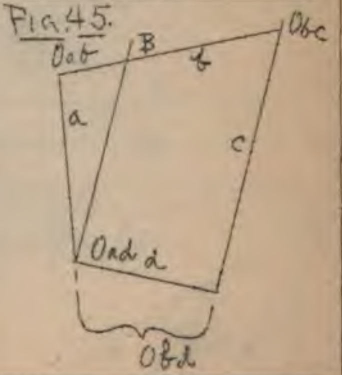

Fig. 45.

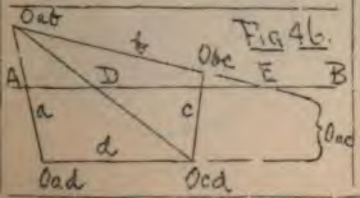

Fig. 46.

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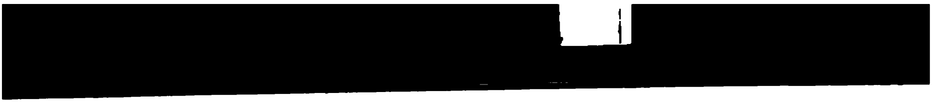
Let r and r′ = the radii of the given steps.
" R and R′ = the radii of the required steps.
" $r+r' = R+R' = a$.
" the velocity ratio of R to R′ = b.

There are two equations between R and R′. $R \div R' = b$, and $R + R' = a$. Combining and solving, it is found that $R' = a \div (1 + b)$, and $R = a - R'$.

Let the case of the open belt be now considered. The formula for length of belt corresponding to the one given for a crossed belt is,

$L = 2\sqrt{d^2 - (R-r)^2} + \pi (R + r) + 2 (R-r)$ (arc whose sine is $R-r \div d$.)

If R and r be changed as before, the term R—r would of course not be constant, and two of the terms of the equation would vary in value, and therefore the length of the belt would vary. The determination of cone steps for open belt therefore becomes a more difficult matter, and approximate methods are almost invariably used. The following graphical approximate method is due to Mr. C. A. Smith, and is given, and the whole subject is fully discussed, in the Transactions of the American Society of Mechanical Engineers, Vol. 10, page 269. Suppose first that the diameters of a pair of cone steps that serve to transmit a certain velocity ratio are given, and that the diameters of another pair that shall serve to transmit some other velocity ratio are required; also that the distance between centres of axes is given. (See Fig. 111.) Locate the pulley centres O and O′, at the given distance apart; about these centres draw circles whose diameters equal the diameters of the given pair of steps; draw a straight line GH, tangent to these circles; at J, the middle point of the line of centres, erect a perpendicular, and lay off a distance JK equal to the distance between centres = C, multiplied by the experimentally determined constant .314; about the point, K so determined draw a circular arc AB, tangent to the tangent line GH to the given pair of steps. Any line drawn tangent to this arc will be the tangent to a pair of cone steps giving the same belt length as that of the given pair. For example, suppose that OD is the radius of one step of the required pair; about O,



with a radius equal to OD, draw a circle; tangent to this circle, and to the arc AB, draw a straight line DE; about O′ and tangent to DE draw a circle; its diameter will equal that of the required step.

But suppose that instead of having one step of the required pair given, to find the other corresponding to it as above, a pair of steps are required that shall transmit a certain velocity ratio $= r$, with the same length of belt as the given pair. Suppose OD and O′E to represent the unknown steps. The given velocity ratio equals r. But from similar triangles $OD \div O'E = FO \div FO'$.

Therefore $r = FO \div FO'$; but $FO = C + x$, and $FO' = x$.

Therefore $r = C + x \div x$, and $x = C \div r - 1$.

From this it will be seen that with r and C given, the distance x may be found, such that if from F a line be drawn tangent to AB, the cone steps drawn tangent to it will give the velocity ratio r, and a belt length equal to that of any pair of cones determined by a tangent to AB.

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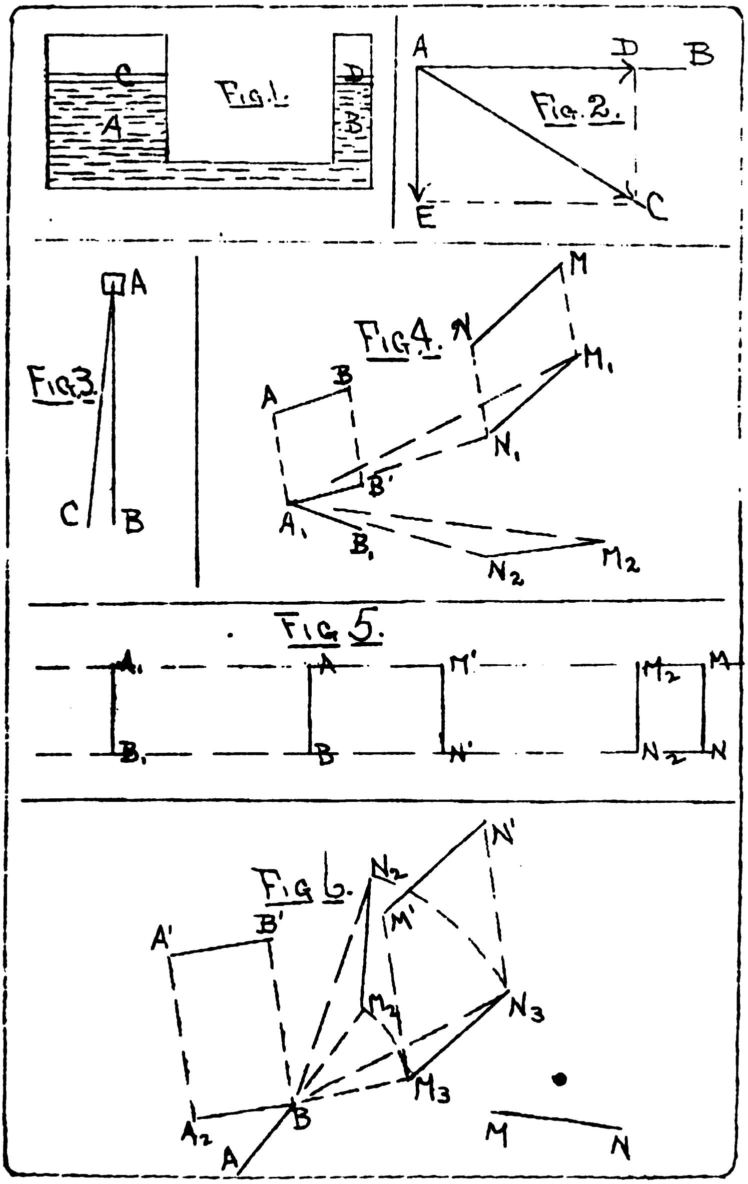
Fig. 1.
C
A
D
B
Fig. 2.
A
D
B
E
C
Fig 3.
A
C
B
Fig 4.
M
N
M₁
A
B
N₁
B'
A₁
B₁
N₂
M₂
Fig 5.
A₁
B₁
A
B
M'
N'
M₂
N₂
M
N
Fig 6.
N₂
N'
M'
A'
B'
M₄
N₃
M₃
B
A₂
A
M
N

Fig. 7.

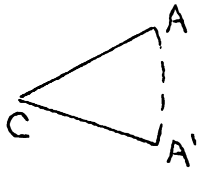

Fig. 8.

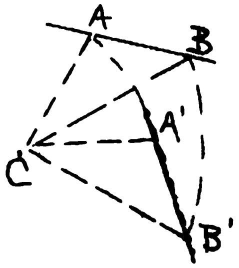

Fig. 9.

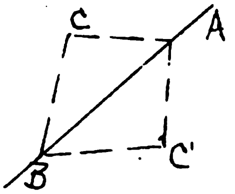

Fig. 10.

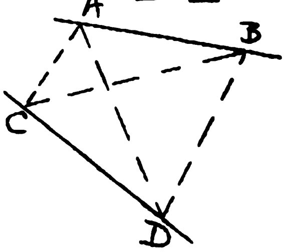

Fig. 11.

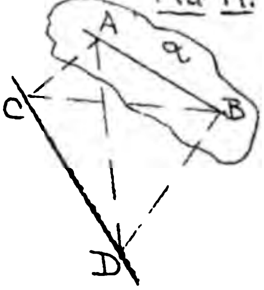

Fig. 12.

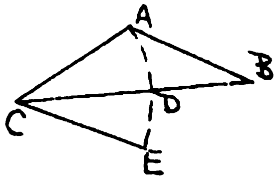

Fig 13.

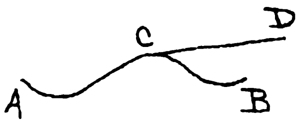

Fig. 14.

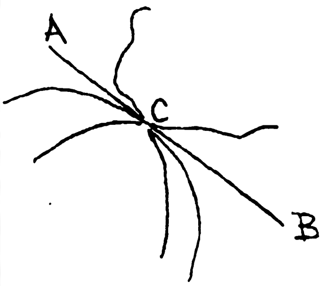

Fig. 15.

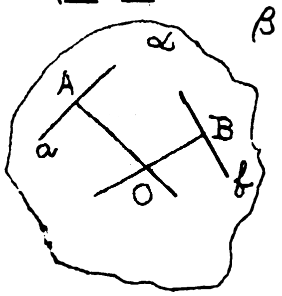

Fig. 16.

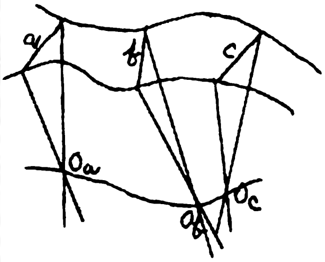

Fig. 17.

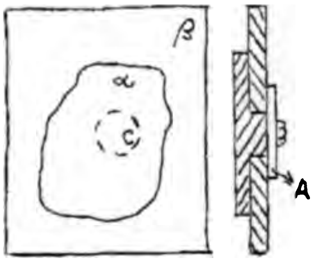

Fig. 18.

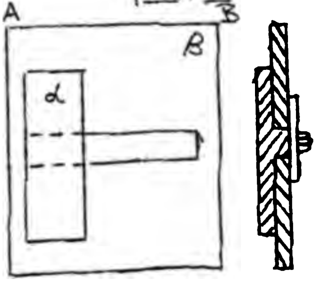

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Fig. 19.

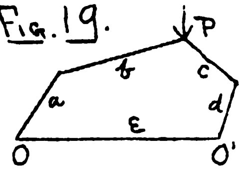


Fig. 20

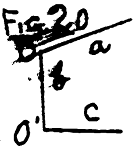


Fig. 22

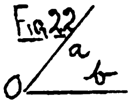
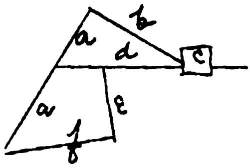


Fig. 21.



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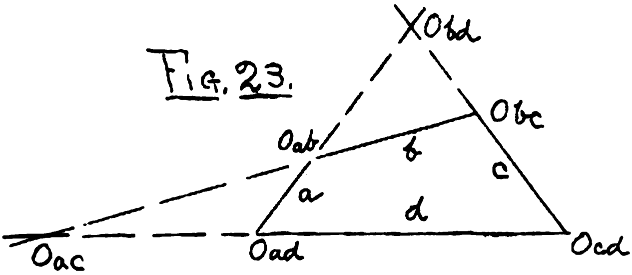

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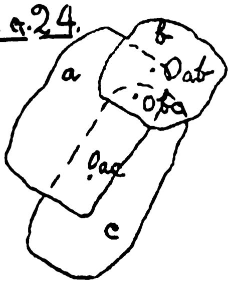

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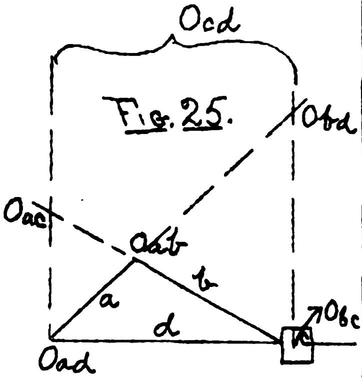

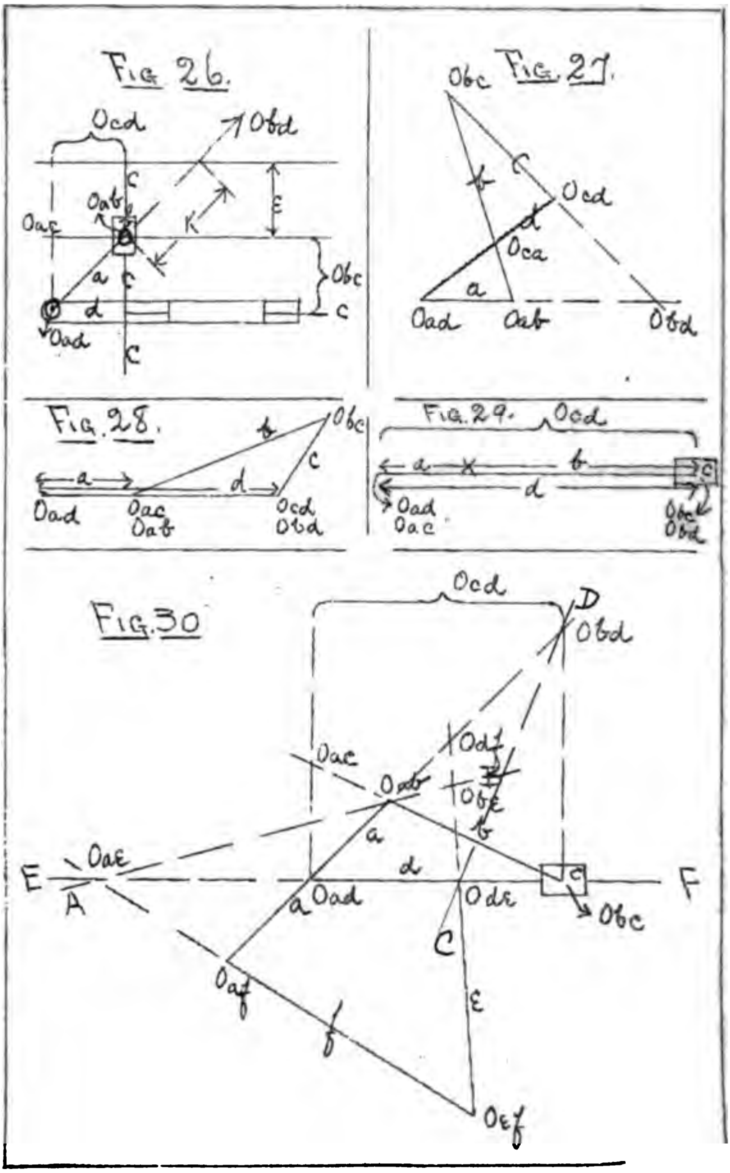
Fig. 26.
Ocd
Obd
Oab
Oac
E
K
Obc
a
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d
Oad
Fig. 27.
Obc
b
c
Ocd
d
Oca
a
Oad
Oab
Obd
Fig. 28.
Obc
b
c
a
d
Oad
Oac
Oab
Ocd
Obd
Fig. 29.
Ocd
a
b
c
d
Oad
Oac
Obc
Obd
Fig. 30
Ocd
D
Obd
Odf
Oac
Oab
Obe
b
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E
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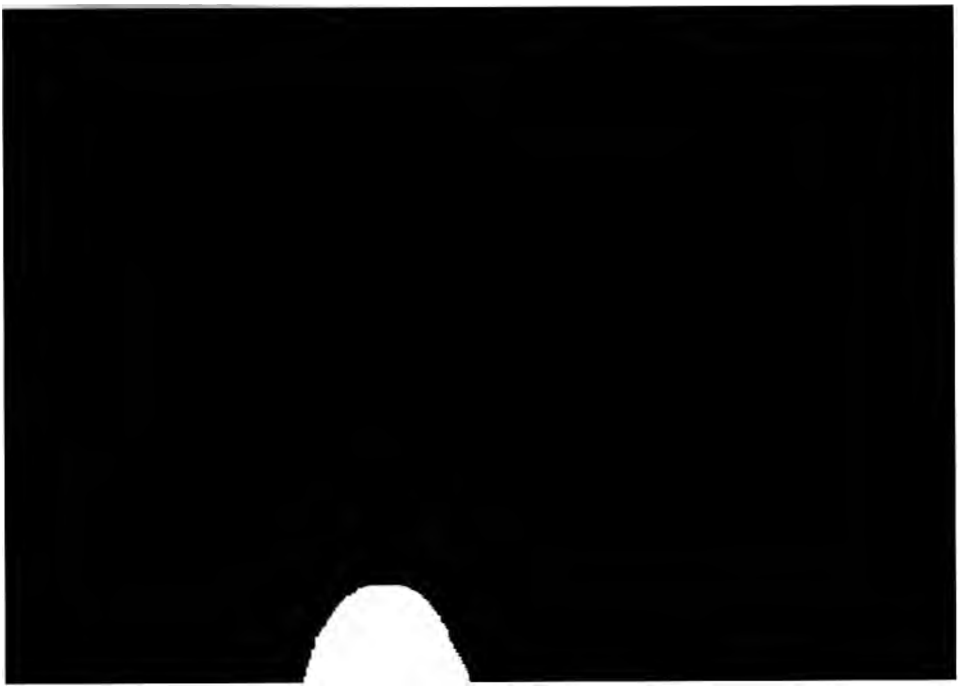


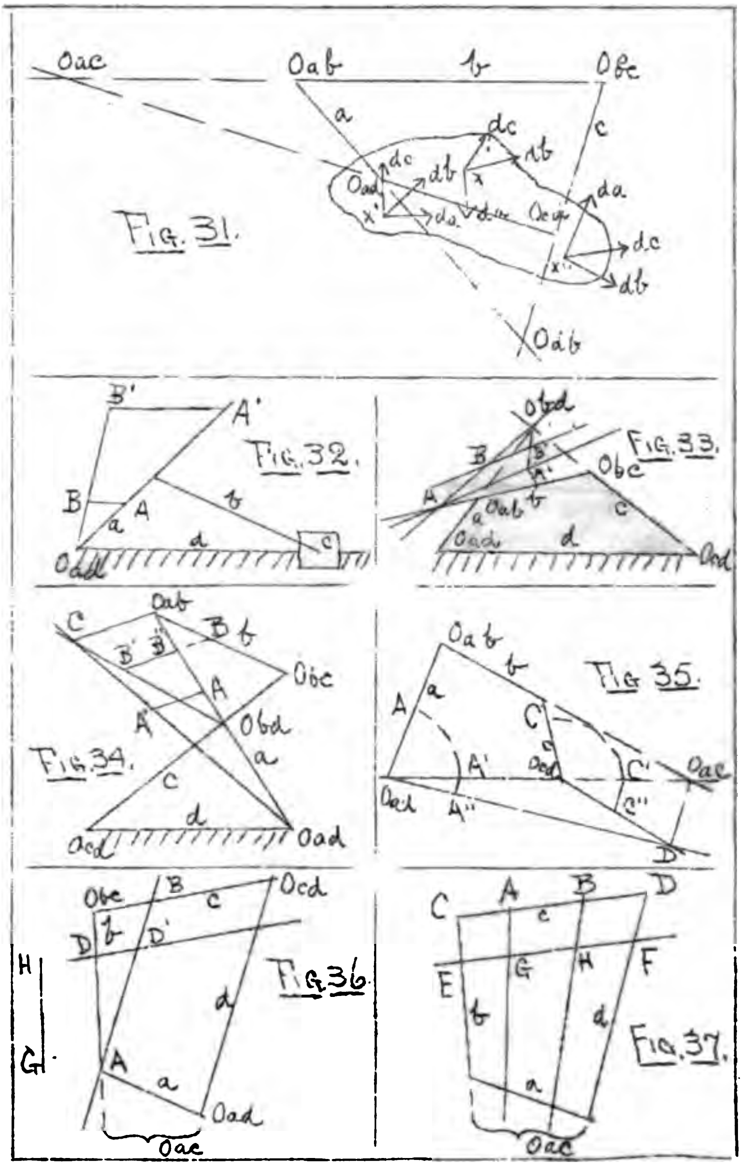


Fig. 31.
Oac
Oab
b
Obc
a
c
dc
db
da
Oad
Ocd
x'
x
x''
Obd
Fig. 32.
B'
A'
B
A
a
b
d
c
Oad
Fig. 33.
Obd
B
B'
A'
A
Obc
Oab
a
b
c
d
Oad
Fig. 34.
Oab
C
B'
B''
B
b
Obc
A
A'
Obd
a
c
d
Ocd
Oad
Fig. 35.
Oab
b
A
a
C
c
A'
Ocd
C'
Oac
Oad
A''
C''
D
Fig. 36.
Obc
B
c
Ocd
D
b
D'
d
H
G
A
a
Oad
Oac
Fig. 37.
C
A
B
D
c
E
G
H
F
b
d
a
Oac

Fig. 57.

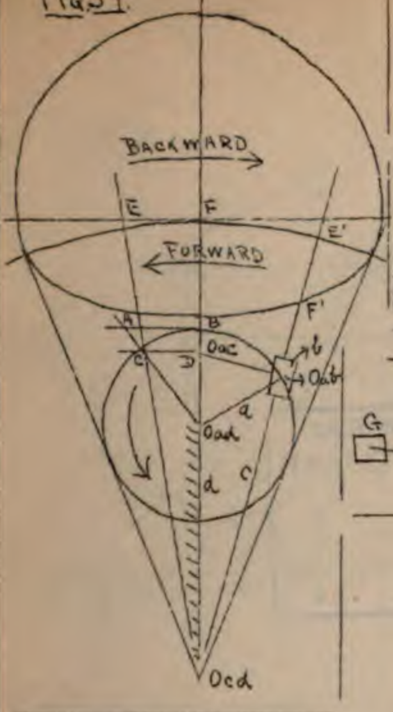
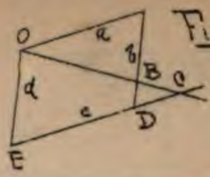


Fig. 58.



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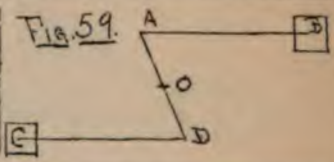

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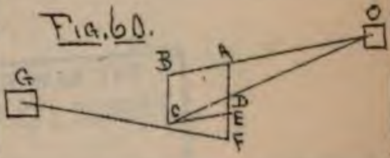

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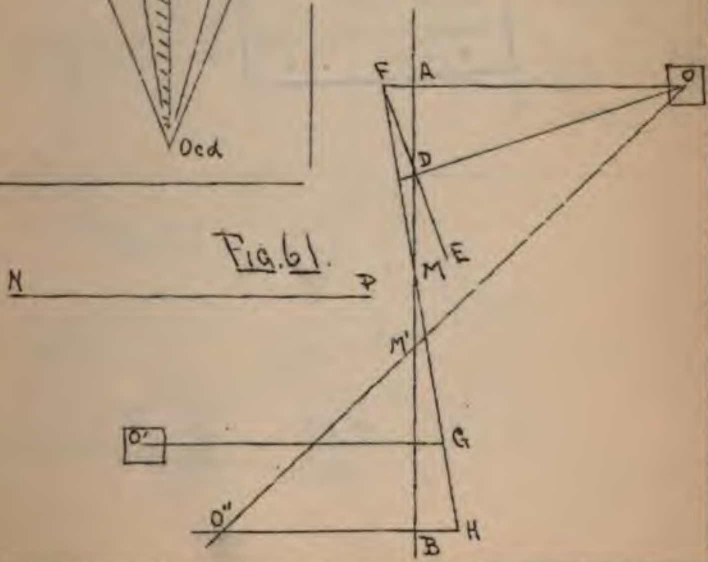


FIG. 47.

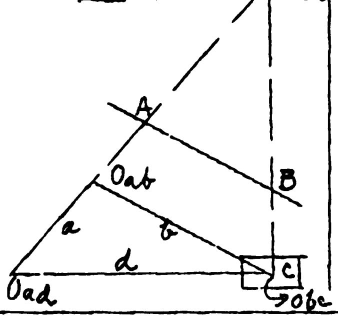
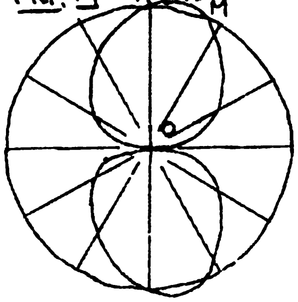


FIG. 49.



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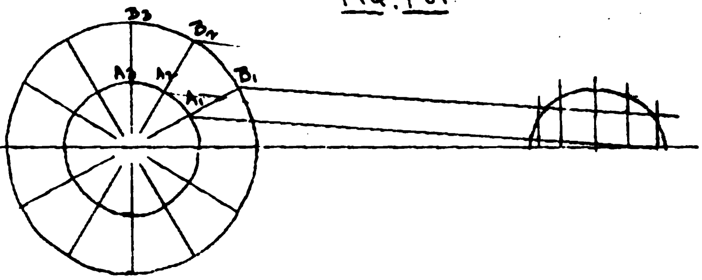


FIG. 51.

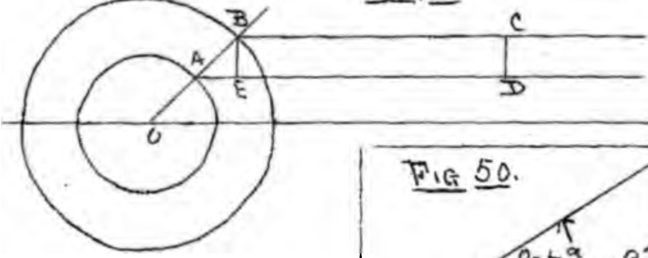
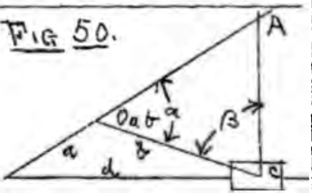


FIG. 50.



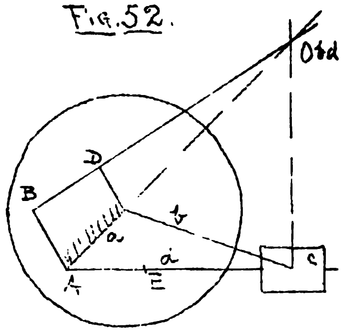
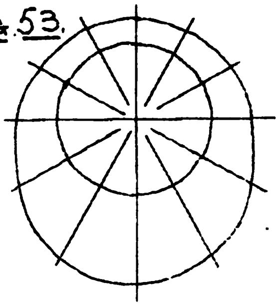


FIG. 52.



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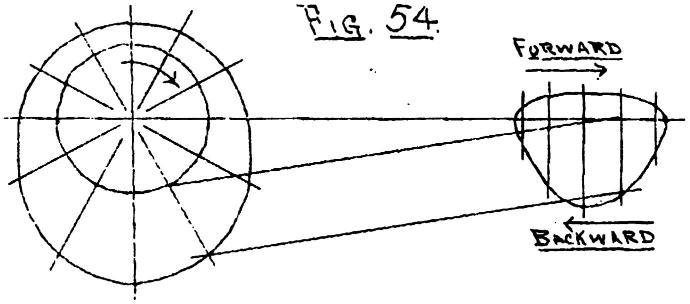

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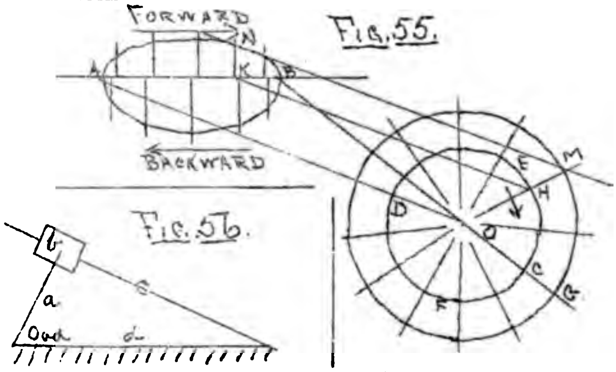

FIG. 55.

FIG. 56.

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FIG. 52.

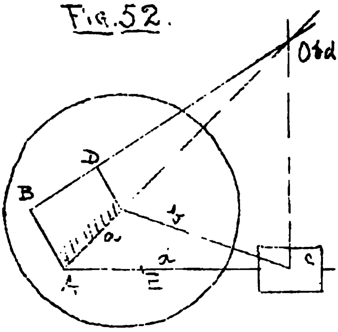


FIG. 53.

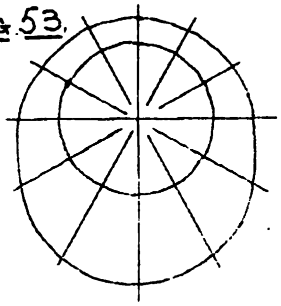
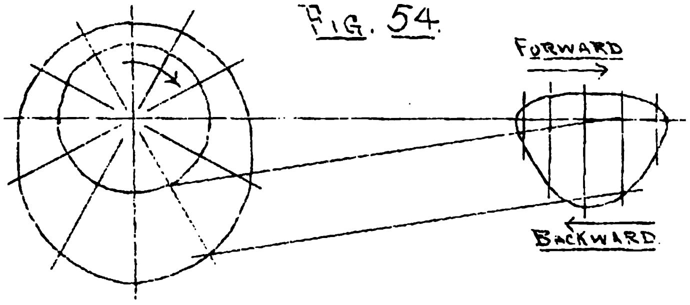


FIG. 54



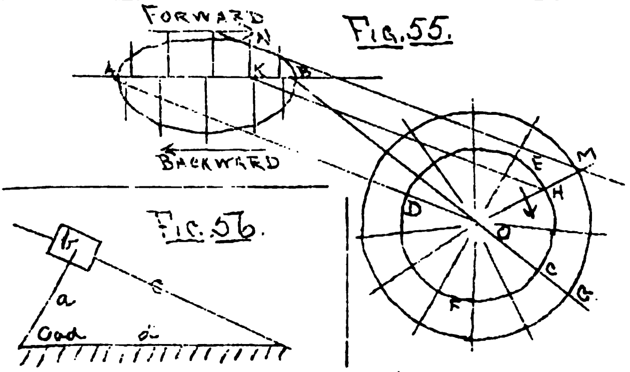


FIG. 55.

FIG. 56.

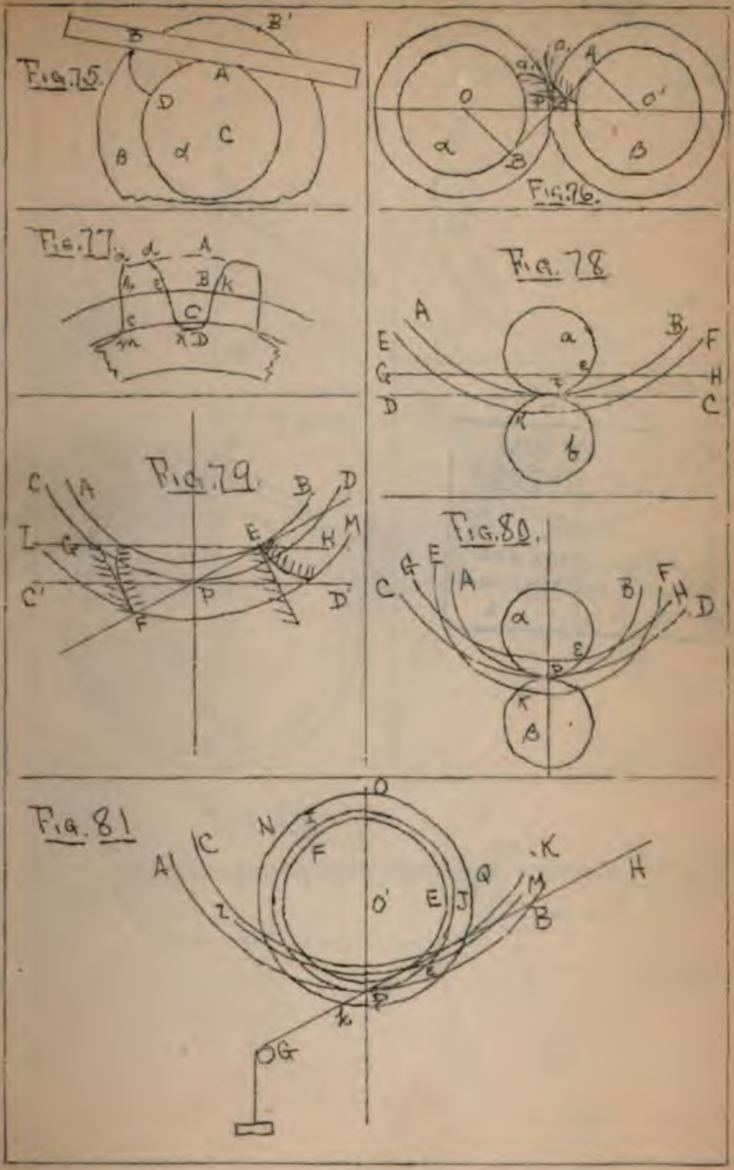


Fig. 75.
Fig. 76.
Fig. 77.
Fig. 78
Fig. 79.
Fig. 80.
Fig. 81

Fig. 62.

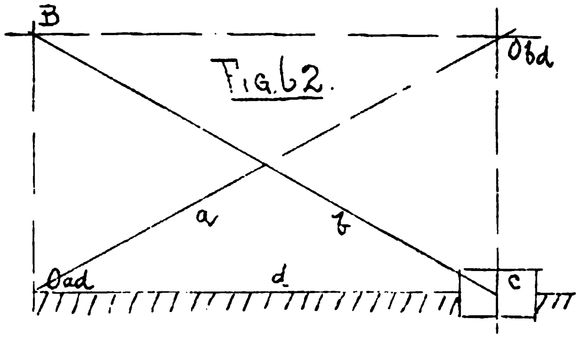


Fig. 63.

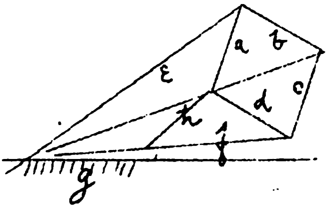


Fig. 64.

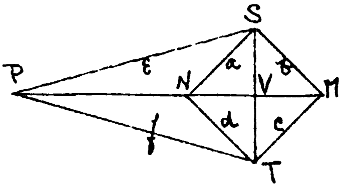
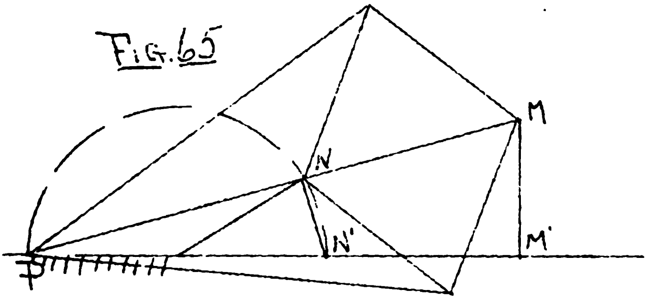
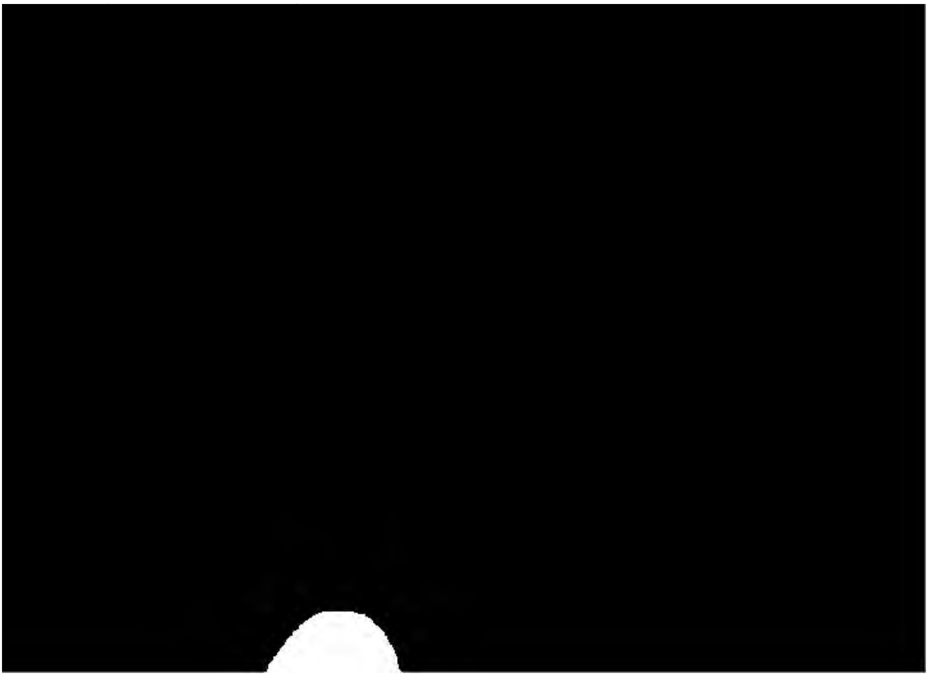


Fig. 65





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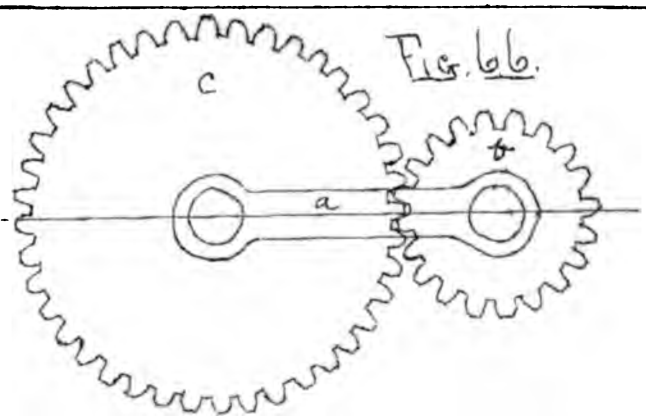

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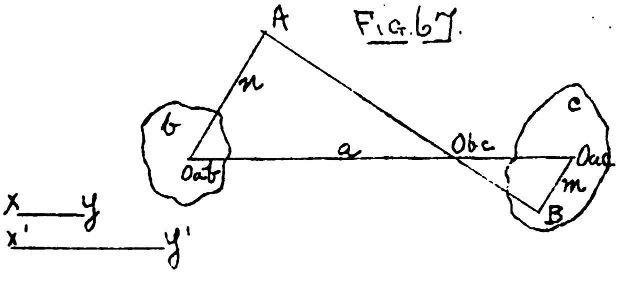

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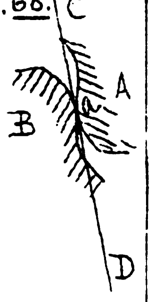

Fig. 69.

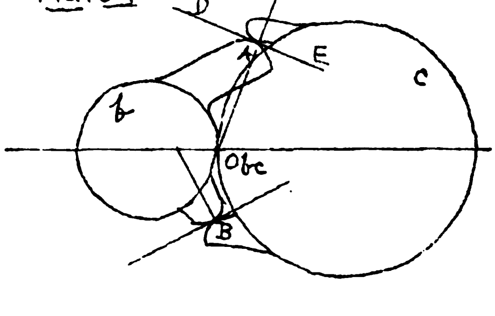

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Fig. 70.

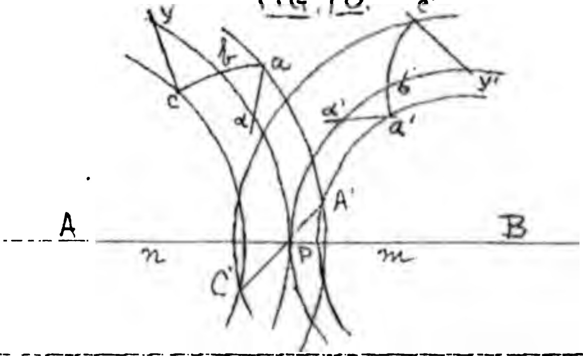


Fig. 71.

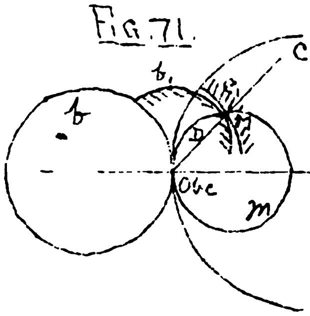


Fig. 73.

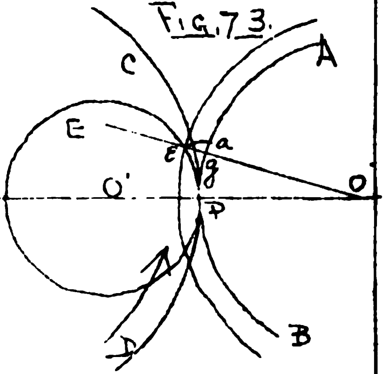


Fig. 72.

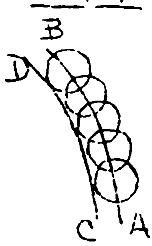
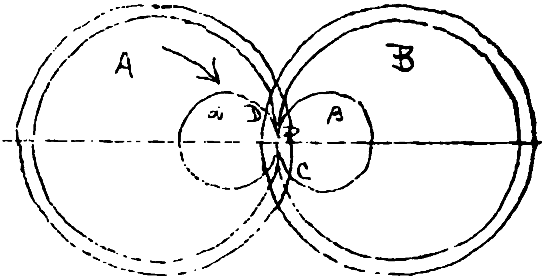


Fig. 74.



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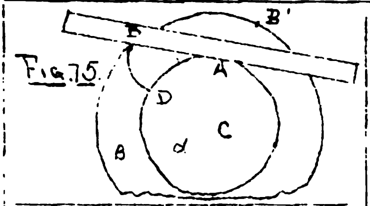


Fig. 75.

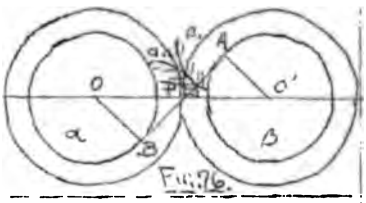


Fig. 76.

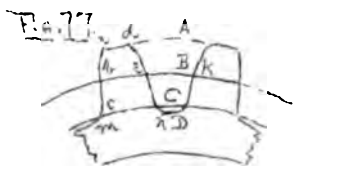
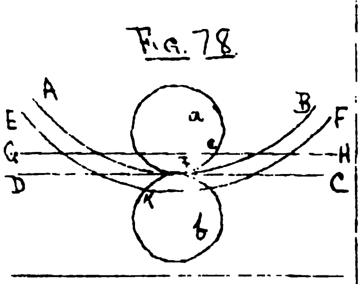


Fig. 77.



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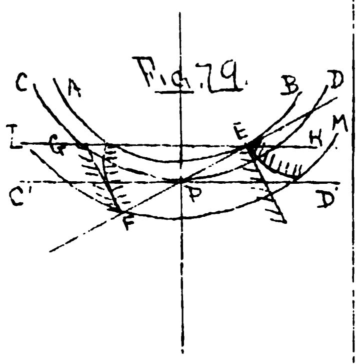

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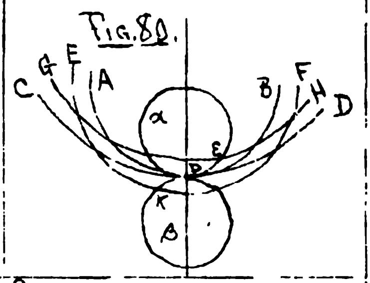


Fig. 80.

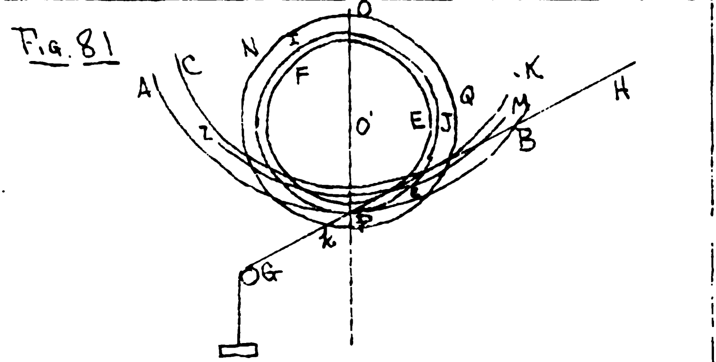


Fig. 81.

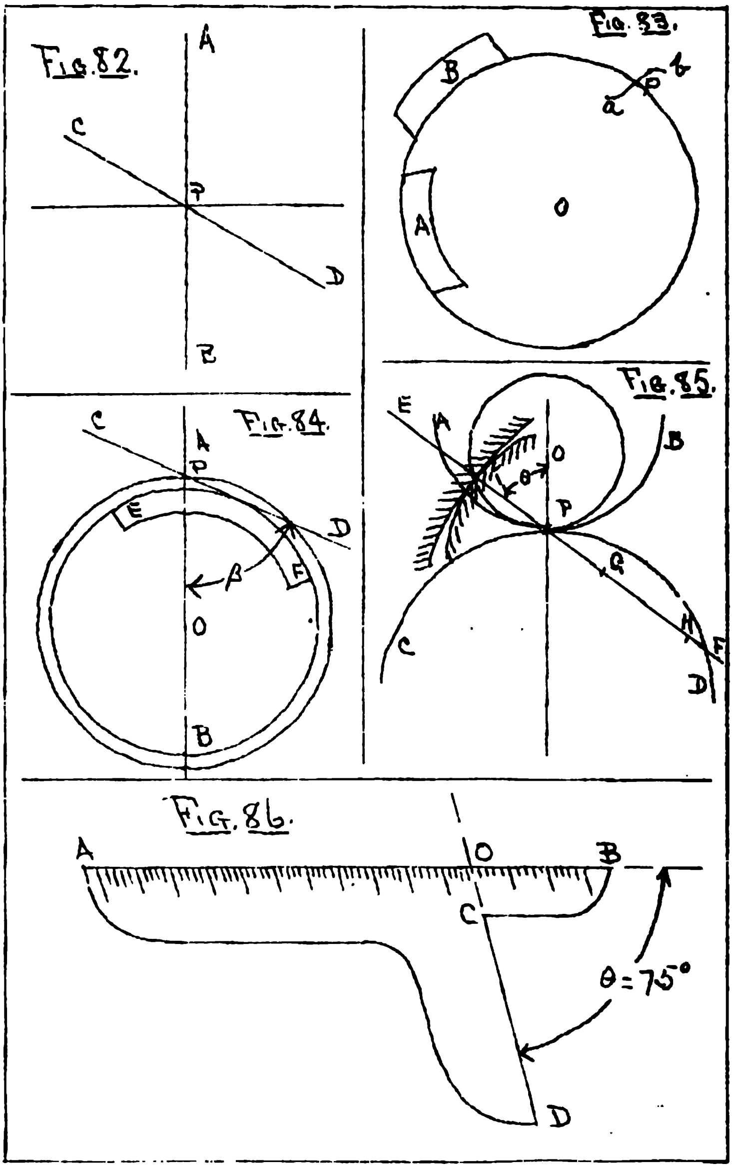


Fig. 82.
A
C
P
D
B
Fig. 83.
B
b
P
a
A
O
Fig. 84.
C
A
P
E
D
β
F
O
B
Fig. 85.
E
A
O
B
θ
P
G
H
F
C
D
Fig. 86.
A
O
B
C
θ = 75°
D

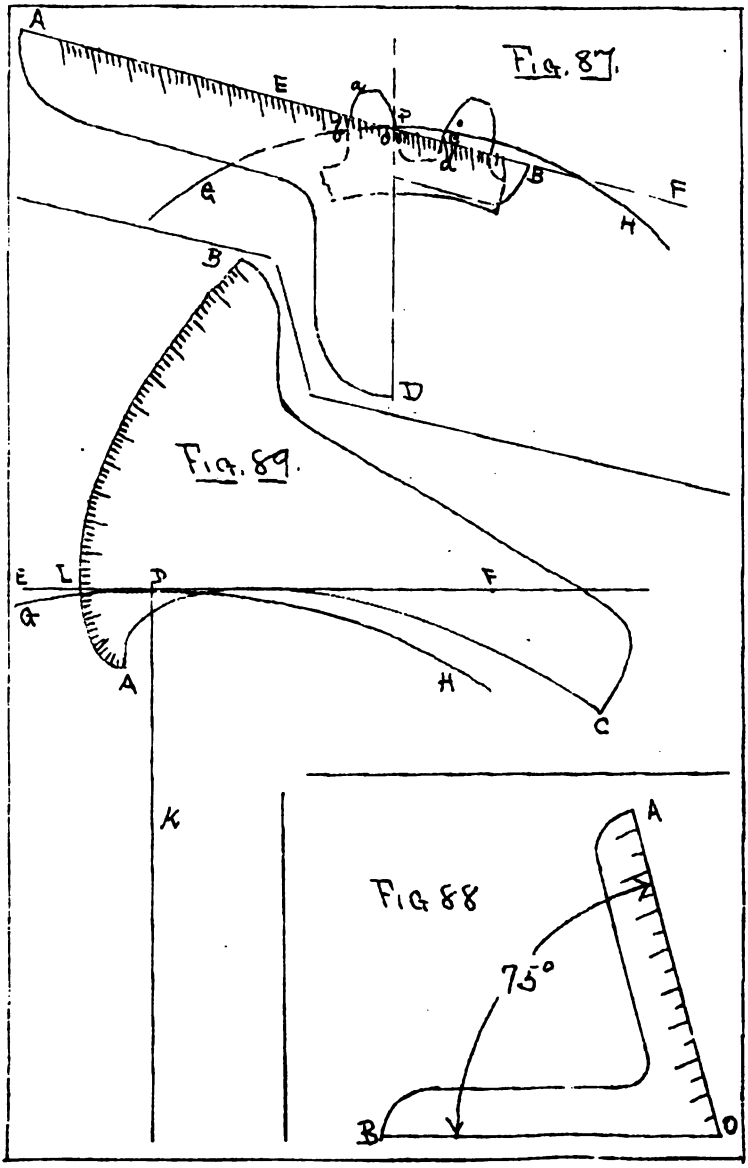


Fig. 87.

Fig. 89.

Fig 88

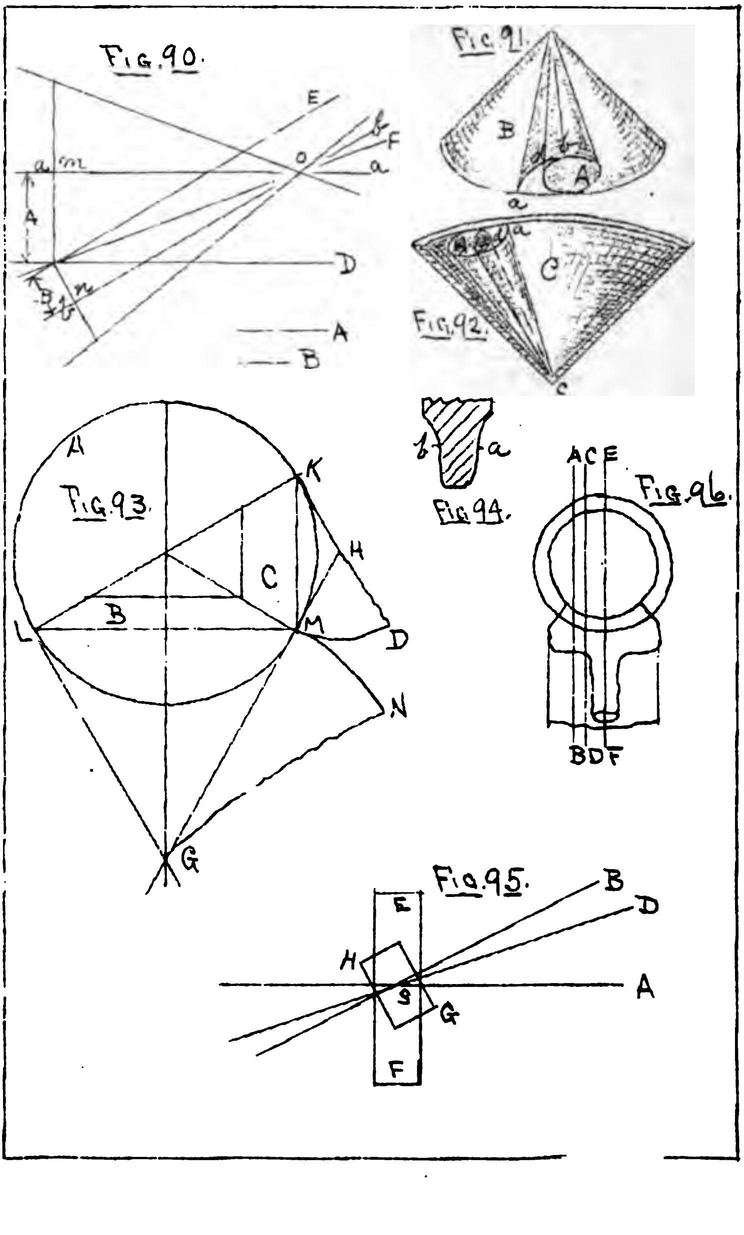


Fig. 90.
E
b
F
o
a
m
A
D
B
b
n
A
B
Fic. 91.
B
d
b
A
a
a
C
Fig. 92.
c
A
Fig. 93.
K
H
C
B
L
M
D
N
G
b
a
Fig. 94.
A C E
Fig. 96.
B D F
Fig. 95.
B
D
E
H
A
S
G
F

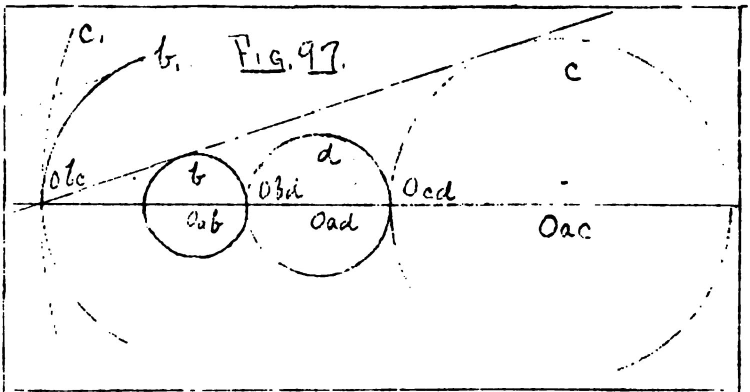


Fig. 97.

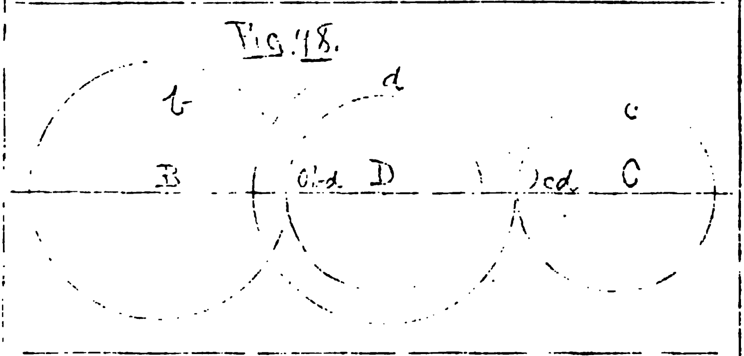


Fig. 98.

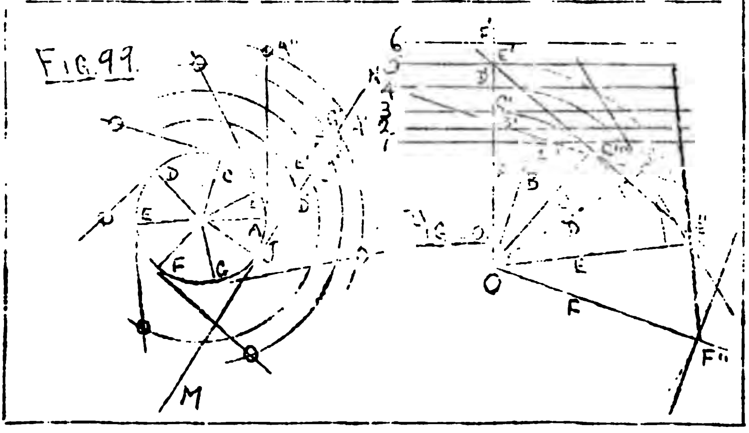


Fig. 99.

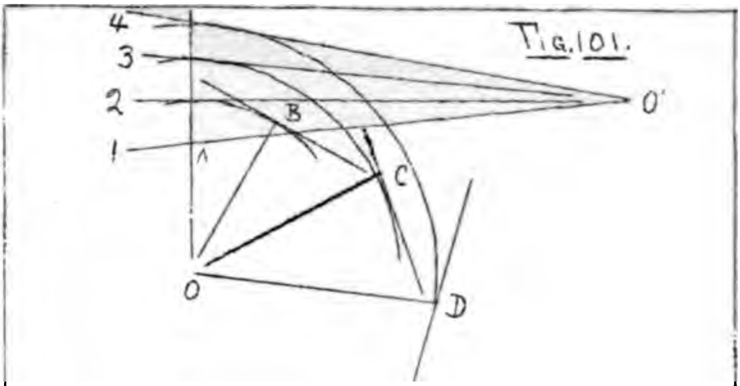


Fig. 101.

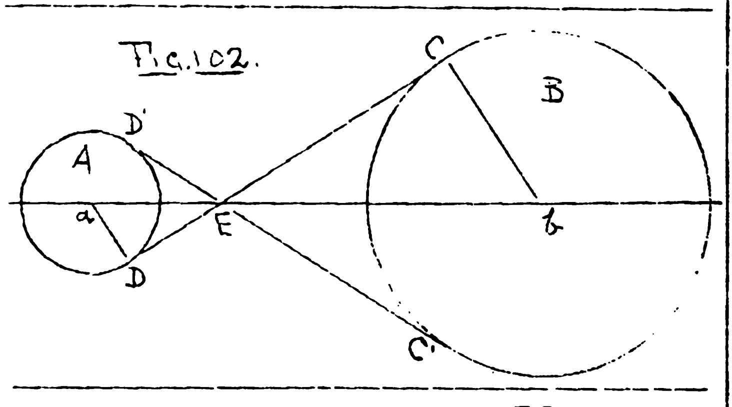


Fig. 102.

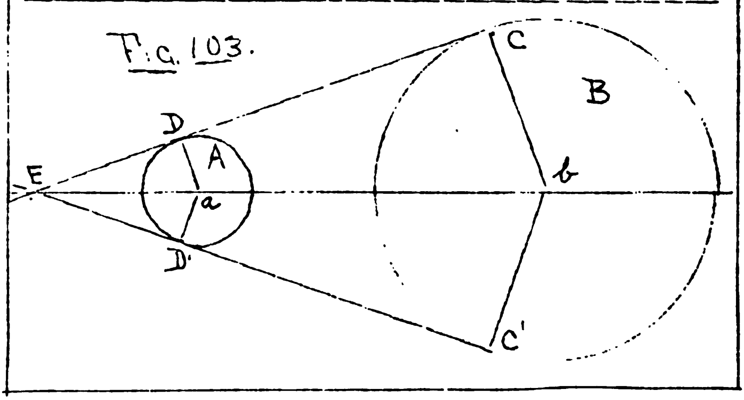
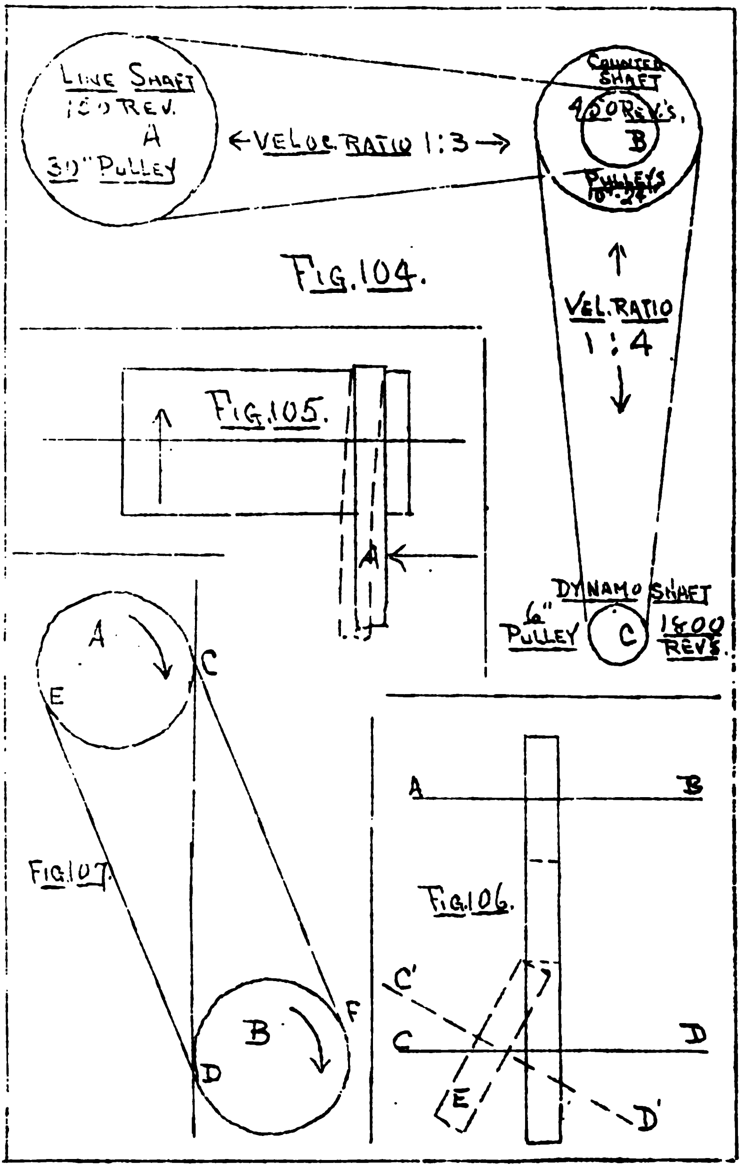


Fig. 103.



Line Shaft
150 Rev.
A
30" Pulley
← Veloc. Ratio 1:3 →
Counter Shaft
450 Rev's.
B
Pulleys
Fig. 104.
Vel. Ratio
1 : 4
Dynamo Shaft
6" Pulley
C
1800 Revs.
Fig. 105.
A
A
C
E
Fig. 107.
B
F
D
A
B
Fig. 106.
C'
C
D
E
D'

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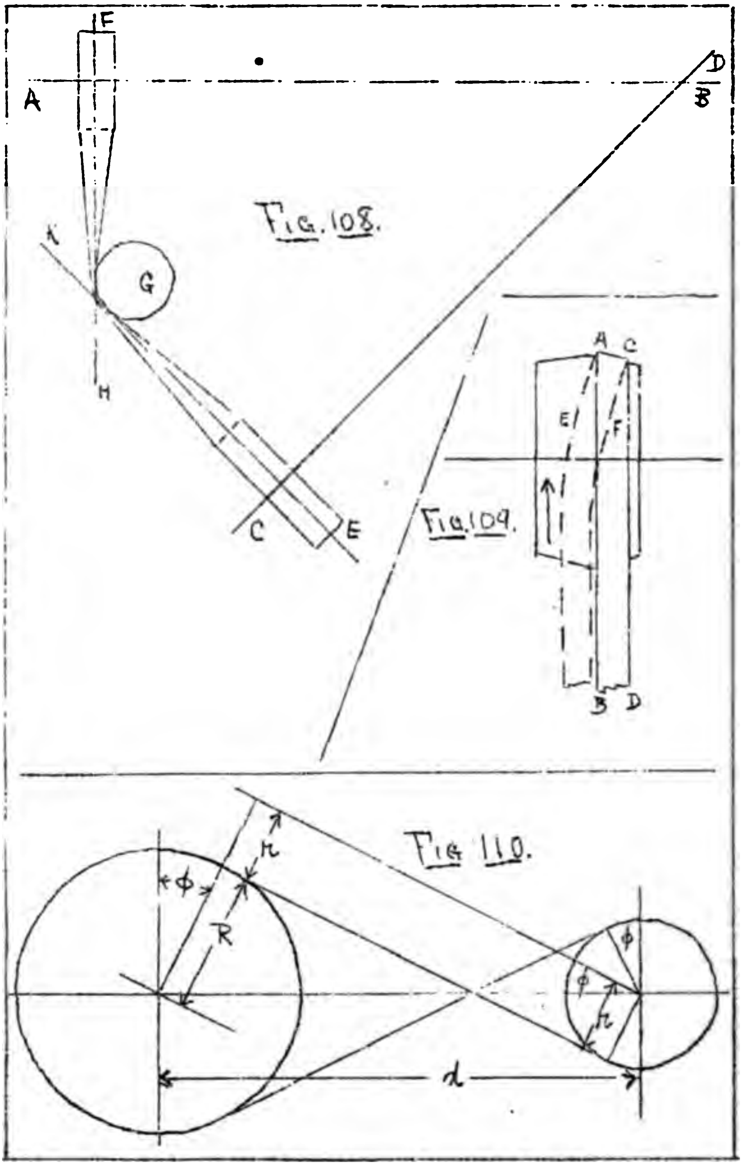


Fig. 108.

Fig. 109.

Fig. 110.

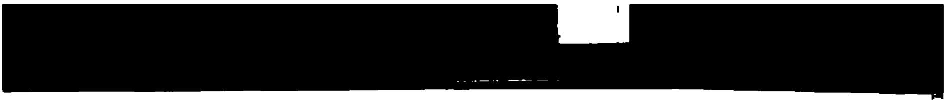
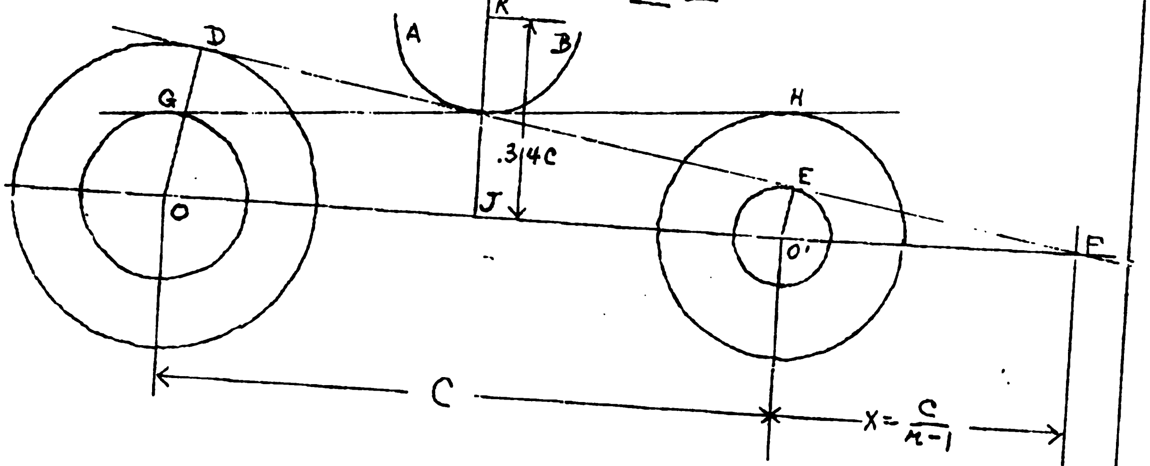


Fig. III.



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NOTES ON LECTURES

IN

MACHINE DESIGN

BY

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ITHACA, N. Y.

1891.

1. Machinery — Construction and design

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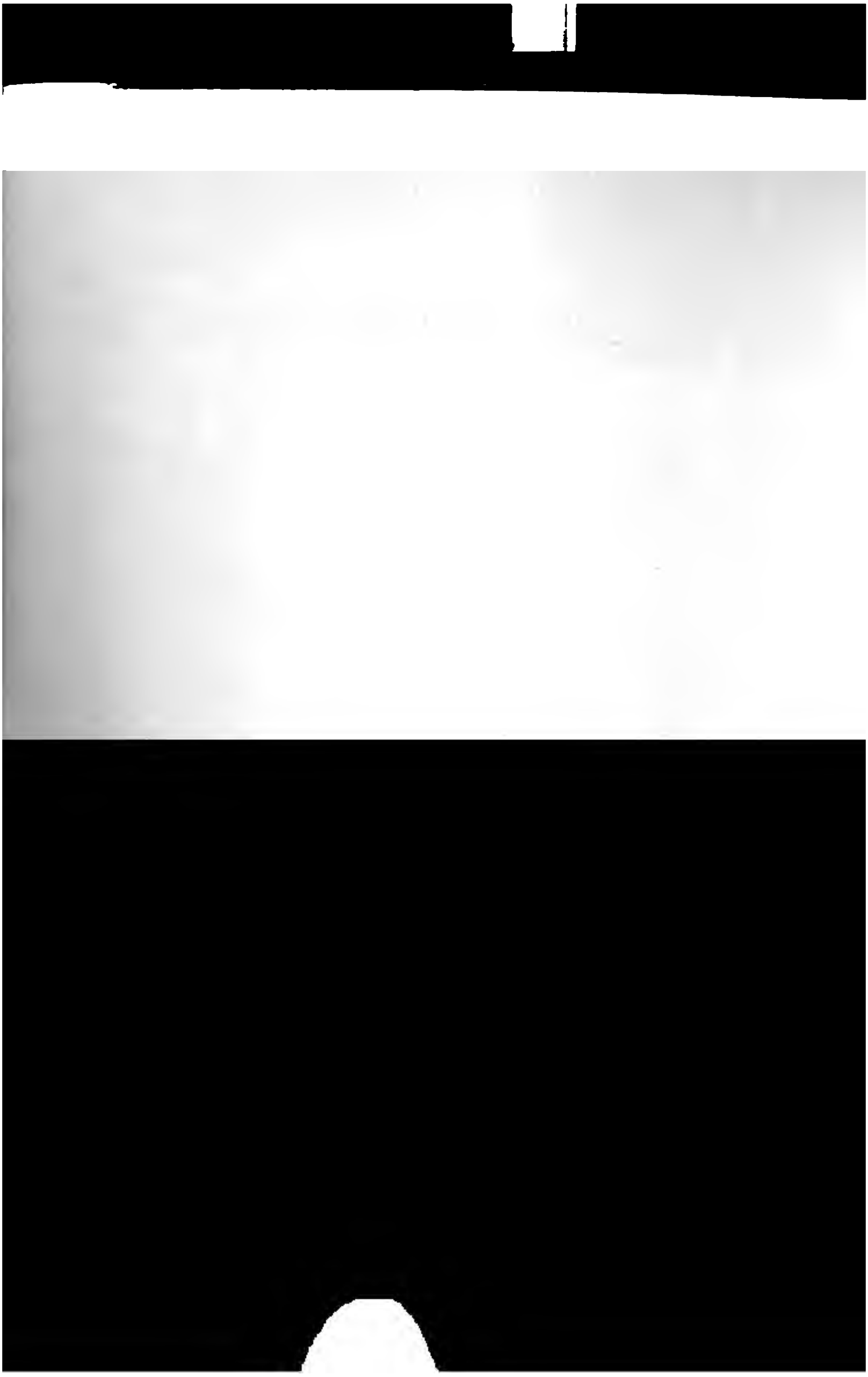
NOTES IN MACHINE DESIGN.

1. A machine is the connecting link between available natural energy and all the desirable results that the various applications of energy can accomplish. The result may be the shaping and surfacing of metals, the cutting and binding, or the grinding of grain, the spinning and weaving of cotton, the printing of books, the generation of electricity, the transportation of freight, or any one of a thousand others, each bringing in an entirely new set of conditions requiring special treatment, and therefore special training of the designer. It is clearly impossible to cover the entire ground, and the designer of machines must be a specialist.

There are, however, certain principles of design that apply to nearly all machines, and there are also certain elements that are the same, or similar in many machines. These admit of general treatment and will be considered first. Afterward machines that may be taken as representing a class, will be selected and worked out more or less completely in detail, for the purpose of illustrating problems in designing, and showing the methods of their solution. It is not expected that the methods given will be necessarily followed, but it is hoped that they may contain enough of suggestiveness so that the student may be led to develop a method of his own, for machine design has much of individuality in it, and almost every designer works in a way peculiarly his own.

2. In general there are four considerations that are of prime importance in the designing of machines: Adaptation, Strength and Stiffness, Economy, Appearance. Adaptation requires that all complexity be reduced to its lowest terms in order that the machine shall accomplish the desired result in the most direct way possible, and with greatest convenience to the operator. Strength and stiffness require that the ma-



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chine parts that are subjected to the action of forces, shall sustain these forces, not only without rupture, but also without that amount of yielding which would interfere with the accurate action of the machine. In many cases the forces to be resisted can be calculated, and then the laws of Mechanics, and the known qualities of constructive materials determine proportions. But in many other cases the forces acting are necessarily unknown; and then appeal must be made to the precedent of successful practice, or to the judgment of some experienced man, until one's own judgment becomes trustworthy by experience.

In proportioning machine parts the designer must always be sure that the stress that is the basis of the calculation or the estimate, is the maximum stress to which the part can be subjected; otherwise the work is lost and the part incorrectly proportioned. For instance, if the arms of a pulley were to be designed from the assumption that they are subjected only to the transverse stress due to the belt tension, they would be found to be absurdly small, because the stresses that result from the shrinkage of the casting in cooling are greater sometimes than those due to the belt pull, and hence must be taken into the account.

The design of many machines is a result of what may be called "machine evolution." The first machine was built according to the best judgment of its designer; but that judgment was fallible, and some part yielded under the stresses sustained; it was replaced by a new part made stronger; it yielded again, and again was enlarged, or perhaps made of some more suitable material; it then sustained the applied stresses satisfactorily. Then it was found that some other part yielded too much under stress, although it was entirely safe from actual rupture; this part was then stiffened, and so the process continued till the whole machine became properly proportioned, as far as the resisting of stress was concerned. Many valuable lessons have been learned from this process; many excellent machines have resulted from it. There are, however, two objections to it: (a) it is

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slow and very expensive, and (b) if any part be given originally an excess of material it is never changed; it is only the parts that yield that are perfected.

The third general consideration is Economy. The attaining of economy does not necessarily mean the saving of metal or labor, although it may mean that. Let a practical case be considered to illustrate: Suppose that it is required to design an engine lathe for the market. The competition is sharp; the profits are small. How shall the designer change the design of the lathes that are on the market so that the profits shall be increased? (a) He may, if possible, reduce the weight of metal used, maintaining strength and stiffness by better distribution. But this must not increase labor in the foundry or machine shop, nor reduce the weight of any part which requires inertia for the absorbing of vibrations that would otherwise prove detrimental. (b) He may design special tools by means of which the labor shall be reduced without reduction of the standard of workmanship. The interest on the first cost of these special tools, however, must not exceed the possible gain from increased profits. (c) He may make the lathe more convenient for the workmen. True economy permits some increase in cost to gain this end. By this it is not meant that elaborate and expensive devices are to be used, such as often come from men of more inventiveness than judgment and experience, and which are usually consigned to the scrap heap after a short time, but that if the parts can be rearranged, or in any way changed so that the lathesman shall select this lathe to use because it is handier when other lathes are available, then economy has been served, even though the cost has been somewhat increased; because the favorable opinion of intelligent workmen means increased sales.

In (a) economy is served by a reduction of metal; in (b) by a reduction of labor; in (c) it may be served by an increase of both labor and material.

The addition of material largely in excess of what is necessary for strength and rigidity, may also sometimes be eco-

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nomical, the object being to provide, by the increased inertia of a stationary part, for the absorption of vibrations that might otherwise injure the machine or its foundation.

Suppose, to illustrate further, that a machine part is to be designed, and either of two forms will serve equally well. The part is to be of cast iron. Call one form A and the other one B. The pattern for A will cost twice as much as for B. In the foundry and machine shop, however, A can be produced a very little cheaper than B. Clearly then, if but one machine is to be built, B should be decided on ; whereas, if the machine is to be manufactured in large numbers, A is preferable. Expense for patterns is a first cost. Expense for work in the foundry and machine shop is repeated with each machine.

In order that economy may be best attained, the machine designer needs to be familiar with all the processes used in the construction of machines ; pattern making, foundry work, forging, and the processes of the machine shop, and must have them constantly in mind, so that while each part designed is not only made strong enough and stiff enough, and properly and conveniently arranged, and of such form as to be satisfactory in appearance, but also is so designed that the cost of construction is a minimum.

The fourth important consideration is Appearance. There is a *beauty* possible of attainment in the design of machines which is always the outgrowth of a *purpose*. Otherwise expressed : A machine to be *beautiful* must be *purposeful*. Ornament for ornament's sake is never admissible in machine design. And yet the striving for a pleasing effect is as much a part of the duty of a machine designer as of an architect.

To indicate something of how the requirements of these general divisions of the subject are met in the practical designing of machines is the object of the following work.

3. On the selection of materials.

The materials ordinarily used for machine parts are :

(a) Crucible steel, also sometimes called "cast steel," or "tool steel."

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(b) Bessemer and Open Hearth steel, called also "machinery steel," or "soft steel."

(c) Wrought iron, also called Malleable iron.

(d) Cast iron.

(e) Malleableized Cast iron, also called Malleable iron.

(f) Steel Castings.

(g) Brass or Bronze.

(h) Babbitt Metal. This name is used to designate all grades of white metal used for lining journal boxes and bearings.

Table of the qualities of materials that affect selection of machine parts:

Material.	*Tensile Strength.*	*Compressive Strength.*	*Resilience or shock Resistance.*	*Shaped for use by*
(a)	very high	very high	medium	forging
(b)	high	high	high	forging
(c)	medium	medium	high	forging
(d)	low	very high	low	casting
(e)	medium	——	high	casting
(f)	high	high	high	casting
(g)	low	——	medium	casting or forging
(h)	very low	——	——	casting

There should always be a film of oil between the metallic bearing surfaces in machines, *i. e.*, surfaces which rub together under pressure; but for various reasons this film often fails, and the metal surfaces themselves come into contact. For this reason the material of bearing surfaces should be selected so that when, under exceptional circumstances they do come into actual contact, heating and "cutting" of the the surfaces shall not result. It is necessary, therefore, to

know what metals will run together safely with defective lubrication.

Wrought iron or steel, either soft or hard, will run safely with wrought iron or steel, either soft or hard, and also with nearly all of the grades of brass and bronze, and babbitt metal. Wrought iron or steel on cast iron is to be avoided if the velocities and pressures are high and the failure of the lubricant possible. Cast iron upon cast iron runs very satisfactorily indeed.

In order to make clear the reasons that lead to the adoption of certain materials for the different parts of machines, certain typical parts will be selected, and the reasons in each case will be given.

The Cylinder of a steam engine, with its ports and its connected steam chest, is of so complicated form that it would be well nigh impossible to shape it by forging; or if the forging were possible, it would be so expensive that it would be out of the question. The possible materials that may be used for such a cylinder are at once narrowed down to those that are shaped by casting. Brass and bronze would have no advantage over cast iron, and would cost about ten times as much. They are, therefore, out of the question. Steel casting could be used, but the first cost of the material would be somewhat greater, and the cost of working in the machine shop would be very much greater. Additional strength and resilience would be gained, but this is unnecessary, as cylinders for even very high pressures can be made of cast iron that are amply strong and resilient, and yet not of objectionably large dimensions. Moreover, cast iron is one of the very best possible materials for the wearing surfaces of the cylinder and valve seat. Cylinders that are subjected to excessively high pressure, as 300 to 700 pounds per square inch, would perhaps be better made of steel casting, as in the case of the cylinders of pumps for pipe lines, or for supplying hydraulic machinery.

The Piston Rod of a steam engine is of soft steel. The entire force of the steam acting on the piston must be trans-

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mitted to the cross head through the piston rod ; also, since the effective area of the piston on the crank side equals the total area of the piston less the area of the rod, and since the effective area needs to be as large as possible, the rod should be as small as possible. There is also always the liability to shocks ; therefore, since the rod must be small and at the same time strong, and must also be capable of resisting shocks, a material is required of high unit strength and of high resilience. Soft steel is the material that combines these qualities.

A steam engine Cross Head Pin is always made larger than is necessary to safely resist shearing, or springing by flexure, in order to insure the maintenance of lubrication ; cast iron might serve then as far as strength and stiffness is concerned, and in fact is sometimes used. But there is another important consideration ; because of the vibratory motion of the connecting rod on the pin, there is a tendency to wear the pin oval, and then when the boxes are "keyed up," they will bind when the rod is in its greatest angular position, if it is properly adjusted when the rod is on the centre line of the engine. Because of this it is desirable to reduce the wear to a minimum, and this points to the selection of a hard material. Hardened tool steel might be used, but it is more expensive than soft steel or wrought iron, and there is the danger of hidden cracks resulting from the hardening that may result in accident. If soft steel be casehardened, it will combine a hard surface that will resist wear well with a soft resilient core that is free from the danger of cracks. Wrought iron casehardened might be used, but wrought iron is not so good for a journal as soft steel, because, from the method of its manufacture, it has streaks of cinder in its surface, and lacks the homogeneity of the steel, and is therefore harder to make, and to keep truly cylindrical. It therefore should not be used where perfection of bearing and accuracy of movement are essential.

The Connecting Rod of a steam engine is subjected to the alternate tension and compression that results from the pres-

sure on the piston, and also to a flexure stress that is due to its vibratory motion. These stresses are very severe, and here too there is the liability of shocks occurring. The material of the rod should be strong and resilient, and soft steel would naturally be selected, since it is a forgable form. But there is another important thing to be considered. The rod is to be finished, and wrought iron is much more cheaply worked in the machine shop than soft steel, and the expense of forging is also much less. The lack of homogeneity is of no importance here, as no part of the rod is a bearing surface. Many connecting rods are made of steel casting and finished by painting. This makes a cheaper rod, but there is always the danger of hidden cracks due to the excessive shrinkage from cooling, or of "blow holes" that do not show on the surface, which may weaken the rod enough to cause accident.

The Cross Head of a steam engine is composed of two parts, (a) that which serves to transmit the pressure from the piston rod to the cross head pin, and (b) that which engages with the guide to produce rectilinear motion parallel to that of the piston. The stresses on (a) are severe, and it is also liable to severe shock, and hence it must be of strong, resilient material; the stresses on (b) however are not very great, but it must be of material that will run well with the guide, which is usually of cast iron, being a part of the engine bed. The cross head may be made of materials as follows: (a) may be made of wrought iron, or soft steel and forged, and (b) may be of cast iron bolted to (a), or the whole cross head may be made of cast iron, the part (a) being made enough larger than before so as to be sufficiently strong; or the cross head may be made a casting of steel and a "shoe" or "gib" of cast iron or brass may be added to provide a proper surface to run in contact with the guide.

The Crank Pin of a steam engine is subjected to the same stress as the cross head pin, and the velocity of rubbing surface is very much greater, hence the tendency to wear is greater. The wear in this case is the same at all points of

the circumference of the pin, and therefore does not interfere with the correct adjustment of the boxes; hence there is not so great necessity for keeping the wear a minimum value; a good journal surface is necessary, and soft steel is used without casehardening.

The Main Shaft of a steam engine needs to be strong and rigid to resist the combination of severe stresses that comes upon it, *i. e.*, the torsional and transverse stress from the connecting rod, and the transverse stress due to the weight of the fly-wheel and the belt tension. It must also afford a good journal surface, and for these reasons it is made of soft steel.

The function of the Fly-Wheel of a steam engine is to adapt a varying effort to a constant resistance, and it does this by absorbing and giving out energy periodically by virtue of its inertia, which is proportional to its weight; it therefore needs, above all things, to be heavy; it also needs to be able to resist the bursting tendency of the centrifugal force due to its rotation. The most suitable material is therefore that which gives the greatest weight in the required form, with the required strength, for the least money, and cast iron best fulfills these requirements.

An engine Bed, or Frame, when it is in one piece, is of cast iron, and the reasons are obvious; its form is complex, and could only be produced by casting. Weight is not objectionable, but rather an advantage, since it absorbs vibrations. Cast iron is amply strong, and affords good wearing surfaces for the cross head guides. Wrought iron is used for engine beds where vibrations are of no importance, as in the locomotive, and where lightness and compactness are very desirable, as in some marine engines. The beds of some of the large roll train and blowing engines are built up of wrought and cast iron.

The journal bearings, or boxes for the cross head pin, the crank pin, and the journals of the main shaft, are usually made now of cast iron or brass, with a babbitt metal lining, because, first, good babbitt metal (tin 80, copper 10, antimony

10) is found to be a better bearing metal than brass, *i. e.*, it runs with less tendency to heat; and second, in the case of the cutting out of the surface, the babbitt lined box is far more quickly and cheaply renewed than the solid brass box.

The eccentric and its strap are almost invariably made of cast iron, because they are forms that are forged with difficulty, and the cast iron affords ample strength and excellent wearing surfaces. The eccentric rod, on the other hand, would be cumbersome and ugly in appearance if it were made of cast iron and given sufficient strength. It is a form that may be easily either forged or cast, and is made of forged wrought iron or steel, or of cast streel, or of malleablized cast iron. Rocker arms also, when they are used, require to be of a resilient material, and when of simple form may be forged of wrought iron or steel; and when of more complex form, may be of malleable cast iron or steel casting. The valve is usually of somewhat complex form, and needs to wear well with the cast iron valve seat, and so is almost invariably of cast iron.

Considerations similar to those above apply to the selection of proper material for the parts of machine tools. Thus, in the case of a lathe, the bed, legs, head and tail stock, cone, gears, etc., are of cast iron, because they are all forms that are most cheaply and satisfactorily produced by casting, and the cast iron affords the required strength and stiffness, and satisfactory wearing surface, where they are required. Such parts as lead screws, feed rods, and other parts that are subjected to some considerable stress, and have great length relatively to their lateral dimensions, are made necessarily of wrought iron or steel. Many of these parts may be finished in the machine shop, directly from merchant bar stock, and so the expense of forging may be saved.

The material for the parts of planing, milling and drilling machines are determined from exactly similar considerations.

Spindles, however, require special attention. In lathes, milling and grinding machines, the accuracy of the work produced depends largely upon the accuracy of the spindle.

The vital point is therefore to maintain this accuracy, *i. e.*, to prevent wear, as far as is possible. It would seem then that hardened tool steel would be the best material. But since only a very small amount of stock can be removed by the grinding machine after the piece is hardened, the spindle must be roughed out very nearly to size before it is hardened; this involves a very considerable expense, and there is danger that it may crack in hardening, or spring so as not to hold up to finish, in which case the loss is large, and it is found that the risk cannot be taken. The next best thing is to specify machinery steel that is high in carbon (say .4%), and to use this harder material for the spindle without hardening. In milling machines, and in some lathes the main spindle box is solid, of tool steel hardened and ground (the risk of loss being less in this case), and the spindle as before is of .4% carbon machinery steel. The wear is thus reduced almost to a minimum; and the possibility of wear after long use is provided against by making the bearing taper and providing end adjustment. The spindles of very large lathes are made of cast iron, because forged material would be too expensive. The wear is reduced by making the journals very large.

In the steam or hydraulic riveter, the main frame that supports the cylinder, and carries the guide for the moving die, may be of any reasonable size, and therefore can be made strong enough to resist even the very great stresses that come upon it if the material used is cast iron. But the "stake," the member that carries the stationary die, must resist exactly the same stresses that come upon the main frame, and must also be small enough so that small boiler shells, and even flues, can be lowered over it to be riveted. The "stake" is therefore of forged wrought iron or steel, or else a steel casting.

Suppose that in a machine there is need of a gear and pinion whose velocity ratio is 8 to 1 and that the force transmitted is large. A tooth of the pinion comes into action eight times as often as a tooth of the gear, and therefore

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would wear out in one eighth of the time, if both were of the same material; then, too, the form of the pinion tooth in most systems of gearing is such that it is much weaker than the gear tooth. From this it will be seen that there is need of a material for the pinion that is not only stronger, but also better able to resist wear. The gear is made of cast iron; if the teeth are cut, the pinion may be made of forged steel; if the teeth are cast and used without "tooling," the pinion may be made a steel casting.

4. General consideration of the form of machine parts that are subjected to different kinds of stress.

Suppose that A and B, Fig. 1, are two surfaces in a machine that are required to be joined by a member that is to be subjected to simple tension; what is the proper form for the member? The stress in all sections of the member at right angles to the line of application of the stress, A B, will be equal and therefore the areas of all such sections should be equal, and so the outlines of the member should be straight lines parallel to A B. The distance of the material from the axis A B has no effect on its ability to resist tension, and therefore there is nothing in the character of the stress that dictates the form of the cross section of the member. The form that is most cheaply produced, both in the rolling mill and the machine shop, is the cylindrical form. Economy, therefore, points to the circular cross section as the best one. So after the required area necessary for safely resisting the stress is determined, it is only necessary to find the corresponding diameter, and it will be the the diameter of all sections of the required member if they are made circular. Sometimes in order to get a more harmonious design it is necessary to make the tension member just considered of rectangular cross section, and this is allowable although it almost always costs more to produce. The thin, wide, rectangular section should be avoided, however, because of the difficulty of insuring a uniform distribution of stress. A unit stress might result from this at one edge that would be greater than the strength of the material, and so the piece would yield by

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tearing, although the average stress might not have exceeded a safe value.

If the stress be compression instead of tension, the same considerations dictate its form as long as it is a "short block," *i. e.*, as long as the ratio of length to lateral dimensions is such that it is sure to yield by crushing instead of by "buckling." A short block, therefore, should have its longitudinal outlines parallel to its axis, and its cross section may be of any form that economy or appearance may dictate. Care should be taken, however, that the *least* lateral dimension of the member be not made so small that it is thereby converted into a "long column."

If the ratio of longitudinal to lateral dimensions is such that the member becomes a "long column," the conditions that dictate the form are changed because it would yield by buckling or flexure, instead of crushing. The strength and stiffness of a long column are proportional to the moment of inertia, (see Church's Mechanics, page 91,) of the cross section about a gravity axis at right angles to the plane in which the flexure occurs. In the case of a long column with "fixed" or "rounded" ends the tendency to yield by buckling is equal in all directions, and therefore the moment of inertia needs to be the same about all gravity axes, and this of course points to a circular section. Also the moment of inertia should be as large as possible for a given weight of material, and this points to the hollow section. The disposition of the metal in a circular hollow section is the most economical one for long column machine members with fixed or rounded ends. This form, like that for tension, may be changed to the rectangular hollow section if appearance requires such change. If the long column machine member be "pin connected," the tendency to buckle is greatest in a plane through the line of direction of the compressive force, and at right angles to the axis of the pins. The moment of inertia of the cross section should therefore be greatest about a gravity axis that is parallel to the axis of the pins, and the forms that should be used are those that are correct for a

beam with a transverse load applied constantly in one direction. Example: a steam engine connecting rod.

When the machine member is subjected to transverse stress the best form of cross section is probably the I section (see *a*, Fig. 2), in which a relatively large moment of inertia, with economy of material, is obtained by putting the excess of the material at the greatest distance from the chosen gravity axis, where it is most effective to resist flexure. Sometimes, however, if the I section has to be produced by cutting away the material at *e* and *d* in the machine shop instead of producing the form directly in the rolls, it is cheaper to use the solid rectangular section *c*, Fig. 2. If the member that is subjected to transverse stress is for any reason made of cast material, as is often the case, the form *b*, Fig. 2, is preferable, for the following reasons: 1st. The best material is almost sure to be in the thinnest part of a casting, and therefore in this case is at *f* and *g*, where it is most effective to resist flexure. 2d. The pattern for the form *b* is more cheaply produced and maintained than that for *a*. 3d. If the surface is left without finishing from the mould, any imperfections due to the foundry work are more easily corrected in *b* than in *a*. Machine members that are subjected to transverse stress, but which are continually changing their position relatively to the force that produces the flexure, must have the same moment of inertia about all gravity axes. As, for instance, rotating shafts that are strained transversely by the force due to the weight of a fly-wheel, or that due to the tension of a driving belt. The best form of cross section in this case is circular, and the hollow section would give the greatest economy of material, but hollow members are expensive to produce in wrought material, which is almost invariably used for shafts, and therefore the solid circular section is used.

Torsional strength and stiffness are proportional to the polar moment of inertia of the cross section of the member which is equal to the sum of the moments of inertia about

two gravity axes at right angles to each other (see Church's Mechanics, page 98). From this it will be clear that the forms in Fig. 2 are not correct forms for the resistance of torsion, but that the circular solid or hollow section or the rectangular solid or hollow section should be used.

The I section, Fig. 3, is a correct form for the resisting the stress P, applied as shown. Suppose now that the web *c* is divided on the line CD, and that the parts are moved out so that they occupy the positions shown at *a* and *b*. The form thus obtained is called a "box section." By making this change the moment of inertia about AB has not been changed, and therefore the new form is just as effective to resist flexure due to the force P as it was before the change. The box section is better able to resist torsional stress, because the change made to convert the I into the box section has increased the polar moment of inertia. Both forms would be equally good to resist tensile stress, and also compressive stress, if both were sections of short blocks. But if they were both sections of long columns, the box section would be preferable, because the moments of inertia would be more nearly the same about all gravity axes.

The framing of machines is almost always subjected to combined stresses, and it will be clear that if the combination of stresses include torsion, flexure in different planes, or long column compression, the box section is the best form for use in the member subjected to the combination of stresses. In fact the box section is by far the best form for the resisting of stress in machine frames. There are also some other reasons beside the resisting of stress that favor its use. 1st. Its appearance is far finer, giving an idea of completeness that is always wanting in the ribbed frames. 2d. The faces of a box frame are always available for the attachment of auxiliary parts without interfering with the perfection of the design. 3d. The strength can always be increased by decreasing the size of the core, and without changing the external appearance of the frame, and therefore without any

work whatever on the pattern itself. The cost of patterns for the two forms is probably not very different; the pattern itself being the more expensive in the ribbed form, and the necessary core boxes adding to the expense in the case of the box form. The expense of production in the foundry, however, is greater for the box form than for the ribbed form, because core work is more expensive than "green sand" work. The balance of advantage is very greatly in favor of box forms, and this is now being recognized in the practice of the best designers of machinery.

To illustrate the application of the box form to machine members, let the table of a planer be considered. The cross section is almost universally of the form shown in Fig. 4. This is evidently a form that would yield easily to a force tending to twist it, or to a force acting in a vertical plane tending to bend it. Such forces may be brought upon it by "strapping down work," or by the support of heavy pieces upon centres. Thus in Fig. 5 the heavy piece E is supported between the centres and to support the piece properly the centres need to be screwed in with a considerable force; this causes a reaction that tends to separate the centres and to bend the table between C and D in a curve that is convex upward. As a result of this, the Vs on the table no longer have a bearing throughout the entire surface of the guides on the bed, but only touch near the ends and the pressure is concentrated upon small surfaces, the lubricant is squeezed out, the Vs and guides are "cut", and the planer is rendered incapable of doing accurate work. If the table were made of the box form shown in Fig. 6, with partitions at intervals throughout its length, it would be far more capable of maintaining its accuracy of form under all kinds of stress and would be more satisfactory for the purpose for which it is designed.

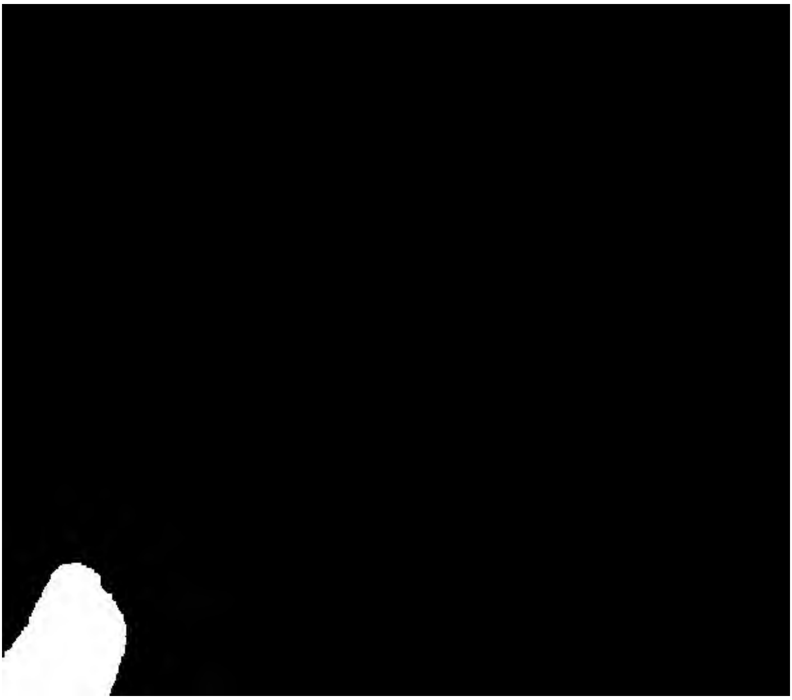
The bed of a planer is usually of the form shown in section in Fig. 7, the side members being connected by "cross girts" at intervals. This is evidently not the best form to

resist either flexure or torsion and both of these kinds of stress may act upon the planer bed either by reason of improper support or because of changes in the form of the foundation, as will be explained in a succeeding section. If the bed were made of box section with cross partitions it would resist stresses far better. Holes could be left in the top and bottom that would admit of the supporting of the core in the mould; would serve for the removal of the core sand; and would render accessible the gearing and other mechanism that is supported inside of the bed.

This same reasoning applies to lathe beds. They are strained transversely by force that tends to separate the centres, as in the case of "chucking;" torsionally by the reaction of a tool that is cutting the surface of a piece of large diameter; and both torsion and flexure may result, as in the case of the planer bed, from an improperly designed or yielding foundation. The box form would be the best possible form for a lathe bed; some difficulties in adaptation, however, have prevented its extended use as yet.

These examples illustrate principles that are of very broad application in the designing of machines.

It often occurs that in machines there is a part that projects either vertically or horizontally and sustains a transverse stress; it is a cantilever in fact. If the transverse stress is the only one and the thickness is uniform, the outline for economy of material is parabolic. (See Church's Mechanics, page 341). In such a case however, the curve that is the outline of the member should start from the point of application of the force, and not from the extreme end of the member, as in the latter case an excess of material would be used. Thus in (1) Fig. 8, P is the extreme position at which the force can be applied and the parabolic curve (a) is drawn from the point of application of P. The end of the member is supported by the auxiliary curve (c). It will be seen that the curve (b) drawn from the end gives an excess of material. The curves (a) and (c) may be replaced by a single continuous



curve as in (3) Fig. 8, or a tangent may be drawn to (a) at its middle point as in (2) Fig. 8, and this straight line used for the outline. The excess of material being slight in both cases.

Most of the machine members of this kind however are subjected also to other stresses; thus the "housings" of planers have to resist torsion and side flexure. They are then very often supported by two members of parabolic outline and, to insure the resistance of the torsion and side flexure, these two members are connected at their parabolic edges by a web of metal that really converts it into a box form.

Machine members of this kind may also be supported by a brace as in (4), Fig. 8. The brace is a compression member and may be stiffened against buckling by a "web" as shown, or by an auxiliary brace.

5. Machine Supports.

The single box pillar support is best and simplest for machines whose size and form admit of its use. When a support is a single continuous member, its design is governed by the following simple principles. First, the amount of material in the cross section is determined by the intensity of the load. If in addition to sustaining a load there are vibrations to be absorbed, the amount of material must be increased for this purpose. Second, the vertical centre line of the support should coincide with the vertical line through the centre of gravity of the part supported. Third, the vertical outlines of the support should taper slightly and uniformly on all sides. If they were parallel they would appear nearer together at the bottom. Fourth, the external dimensions of the support must be such that the machine has decidedly the appearance of being in stable equilibrium. The outline of all heavy members of the machine supported must be either carried without break to the foundation, or if they overhang, they must be joined to the support by means of parabolic outlines, or by the straight lines of the brace form.

Thus to illustrate in A, Fig. 9, the first three principles may be fulfilled, but there is the appearance of instability, and it is evidently because the outline of the "housing" overhangs. It should be carried to the foundation without break in the continuity of the metal as in B.

When the support is divided up into several parts some modification of these principles becomes necessary, as the divisions require separate treatment. This question may be illustrated by the consideration of lathe supports. In Fig. 10 are shown three forms of support for a lathe, seen from the end. For stability the base needs to be broader than the bed. In (a) the width of base necessary is determined and the outlines are made straight lines. Then the unnecessary material is cut away on the inside leaving the legs AB and CD which are compression members of correct form. The cross brace F is left to check any tendency to buckle. For convenience to the workmen it is desirable to narrow this support somewhat without narrowing the base. The cross brace coverts the single compression member AB into two compression members and it is allowable to give these different angles with the vertical axis. This is done in (b) and the straight lines GH and HK are blended into each other by a curve at H. (c) shows a common incorrect form of lathe support, the compression members from the cross brace downward being curved. There is no reason for this curved form and it is less capable of bearing its compressive load than if it were straight, and is no more stable than the form (b) the width of base being the same in both cases. Let the lathe supports next be considered from the front. Four forms are shown in Fig. 14. If there were any force tending to move the bed of the lathe endwise the forms (b) and (c) would be allowable. But there is no force of this kind, and the correct form is the one shown in (d). Carrying the foot out as in (a), (b) and (c) increases the distance between supports (the bed being a beam with end supports and the load between) this increases the deflection and the fibre stress

due to the load. If the supports were joined by a cross member as in Fig. 10, they would be virtually converted into a single support, and should then taper from all sides.

If a machine be supported on a single box pillar a change in the form of the foundation cannot bring stress upon the machine that would tend to change its form. If, however, the machine is supported on four or more legs the foundation might sink away from one or more of them and leave a part unsupported, and this might bring a torsional or flexure stress on some part of the machine that might change its form and interfere with the accuracy of its action. But if the machine be supported on three points this cannot occur, because, if the foundation should sink under any one of the supports, the support would follow and the machine would still rest on three points. When it is possible therefore a machine that cannot be carried on a single pillar should be supported on three points. In many cases the machine is too large for a three-point support and then the resource is to make the bed or part that is supported of box section and so rigid that even if some of the legs should be left without foundation to rest upon the part supported shall still maintain its form. In many cases in machine design more supports than are necessary are used. Thus if a lathe have two pairs of legs like those shown in (b) Fig. 10, and these be bolted firmly to the bed, there will be four points of support. But if, as suggested by Professor Sweet, one of these pairs be connected to the bed by a pin so that the support and the bed are free to move relatively to each other about the pin as an axis as in Fig. 13, then this is equivalent to a single support, and so the bed will have three points of support, and will maintain its form independently of any change in the foundation. This is of special importance when the machines are to be placed upon yielding floors as so often occurs.

Fig. 14 shows another case in which the number of supports may be reduced without sacrifice. In A three pairs of

legs are used and there are therefore six points of support; in B two pairs of legs are used and one may be connected by a pin and therefore there will be but three points of support and the chances of the bed being strained from changing foundation has been reduced from 6 in A to o in B. The total length of bed is 12′ and the unsupported length is 6′ in both cases.

Figs. 15 and 16 show correct methods of support for small lathes and planers, and are due to Professor Sweet. In Fig. 15 the lathe "head stock" has its outlines carried to the foundation by the box pillar; (a) represents a pair of legs that are connected to the bed by a pin connection, and instead of being placed at the end of the bed it is moved in somewhat, the end of the bed, being carried down to the support by a parabolic outline. The unsupported length of bed is thereby decreased, the stress on the bed is less, and the bed will maintain its form regardless of any yielding of the floor or foundation. In Fig. 16 the housings instead of resting on the bed as is usual in small planers, are carried to the foundation and form two of the supports; the other is at (a) and has a pin connection with the bed, which being thus supported on three points cannot be twisted or flexed by a yielding foundation.

6. Concerning machine parts that are formed by casting.

When a metal like cast iron or steel cools from a fluid state and solidifies, its structure becomes crystalline, and the lines of crystalization arrange themselves at right angles to the surface from which the flow of heat takes place. These lines of crystalization may be represented by lines as in Fig. 17. Along the line ab the lines of crystalization intersect and the structure is broken up and a line of weakness is the result. If however the corner be rounded as at (c) and the re-entering angle be replaced by a "fillet" as at (d), the direction of the lines of crystalization is changed gradually and the line of weakness is avoided. Whether this theory is correct or not, it is known practically that the line of

weakness does exist under the circumstances described and may be avoided as suggested. The conclusion from this is, that sharp corners and re-entering angles are to be avoided in the design of cast machine members. Appearances also requires the rounded outlines.

Experience points to the conclusion that castings of small cross section shrink more than those of large cross section. To test this conclusion Mr. Thomas D. West made an experiment that he describes in his book, "American Foundry Practice." He cast two bars 14 ft. long from the same iron, and as far as possible made the other conditions of the casting the same for both. Both were of rectangular cross section, one being $4'' \times 9''$ and the other $\frac{1}{2}'' \times 2''$. The total shrinkage of the larger bar was $\frac{7}{8}''$ and of the smaller bar was $1\frac{3}{4}''$. This may possibly be explained as follows, as Mr. West suggests. Since a casting cools from the surface therefore during the cooling the surface will be the coolest part and the heat will increase toward the centre. The external portions are held from their normal shrinkage by the resistance of the hotter internal portions that are not yet ready to shrink as much. This goes on till the surface has reached the temperature of the surrounding atmosphere and stops shrinking ; the hotter portions nearer the centre now try to shrink as they in turn cool down but are prevented by the external part that has stopped shrinking. This action is necessarily greater in large castings than in small ones and therefore the shrinkage is less in the larger ones.

If this theory be correct the internal stresses that are due to shrinkage will increase with the size of the casting. This may accout for the tendency of large castings to crack, especially in re-entering angles. It also follows that castings having thick and thin parts that are attached to each other will shrink unequally and therefore be in a state of internal stress that would render them less able to resist external stresses.

If portions of a casting have large cubic contents, the

fluid shrinkage may be supplied by careful "feeding" through the "riser" in the foundry; but after the surface of the casting has solidified and the riser has "frozen up," the shrinkage goes on and the internal portion, a part of which may still be fluid goes on shrinking and having no further supply to draw upon may become spongy, and weak to resist external stress. The casting therefore might have been stronger if it had been made thinner.

Suppose that it is desired to put a strengthening rib on (A) Fig. 18, and that it is made of the form shown; i. e. thin relatively to A and having parallel sides. B would shrink more than A and this would put shrinkage stresses upon the casting which would be concentrated along the juncture of A and B, because of which it would yield there more easily under external stress. But if the form in Fig. 19 were used the shrinkage stresses would be diminished and distributed, and the casting would be stronger to resist external stresses.

The lessons to be learned from the above are as follows: 1st. All parts of all cross sections of castings for machine parts should be as nearly of the same thickness as possible, to avoid shrinkage stresses and spongy metal with their accompanying weakness. 2d. If it is necessory to have thick and thin parts in the same casting, change of form from one to the other should be as gradual as possible. 3d. Castings should be made as thin as is consistent with strength and stiffness and resistance to vibration, to avoid the stresses due to the shrinkage of large castings. 4th. Since shrinkage stresses always exist, they must always be considered, and the cast members must be designed so as to be able to resist them in addition to the external stresses. This is of the utmost importance.

Special care should be taken with the design of wheels. In a pulley the thin rim tends to shrink more than the heavier arms and the rim is thereby put in tension that may cause its rupture. If the same pulley has a relatively heavy hub the latter will remain fluid until the arms and rim

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have solidified : the tension of the rim will then force the arms into the yet fluid hub and that in turn shrinking will put a tensile stress on the arms that may result in their rupture. In fly wheels with heavy rims the lighter arms tend to shrink away from the rim and are therefore in tension. All very large fly wheels should be cast in sections that are of such form that they are free to yield to shrinkage stresses, and then should be fitted and fastened together.

Steel castings solidify at about 3500°F while cast iron solidifies at about 2500°F. The shrinkage in the former is therefore proportionally greater and far greater care should be taken with the design.

7. Illustrations of the designing of machine frames.

It is required to design the frame of a power punch that shall be able to punch ¾″ holes in ½″steel plates 18″ from the edge. The surface that resists the shearing action of the punch $= \pi \times ¾'' \times ½'' = 1.17$ square inches. The ultimate shearing strength of the material is say 50,000 pounds per square inch. The total force P, applied as in the Fig. 20, that has to be resisted by the punch frame $= 50{,}000 \times 1.17 = 58500$ pounds, say 60,000 pounds. The stresses that come on the section AB, Fig. 20 are (a) a tensile stress $= P$ distributed over the area of the section at AB, and (b) a flexure stress produced by P acting on the lever arm l. This latter tends to compress the metal at B, and to extend it at A (see Church's Mechanics, psge 347).

The unit stress due to tension $= p = P \div F$.

The maximum unit stress due to flexure $= p' = Ple \div I$.

$F =$ the area of the cross section at AB.

$e =$ the distance from the gravity axis to the outer fibre in tension.

$I =$ the rectangular moment of inertia of the cross section at AB about the gravity axis perpendicular to the paper.

The maximum unit stress in the cross section at AB will equal $p + p'$. This may be shown graphically. AB, Fig. 21, represents the dimension AB, Fig. 20. The stresses that

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come on the cross section are represented as ordinates measured from AB as an X axis. G is the location of the gravity axis of the section. The ordinates of the double triangular diagram represent the stresses that come on the section because of the flexure stress. These are negative, i. e., compression at the right of the gravity axis. The constant value of the tension throughout the section $= p$ is represented by the band whose constant vertical dimension is $= p$. This diagram shows that the tension on the inner fiber p' is increased by the uniformly distributed tension p, also that the maximum compression p'' is diminished by p. Unless the gravity axis were very near A the maximum stress in the section would be tension at A, and would be equal to $p + p'$.

The material and form for the frame must first be selected. The form is such that forged material is excluded, and difficulties of casting and high cost exclude steel casting. The material therefore must be cast iron. In many cases the same pattern is required to be used both for the frame of a punch and shear. In the latter case when the shear blade begins and ends its cut the force is not applied in the middle plane of the frame, but considerably to one side, and a torsional stress is thereby brought on the frame. Combined torsion and flexure, are best resisted by members of box form. The frame will therefore be made of cast iron and of box section.

The dimension AB may be assumed so that it appears in good proportion to the reach of the punch, and then the width and thickness of the cross section may be assumed, and from these data the maximum stress in the outer fibre may be determined, and if this is a safe value for the material used, the form will be correct. Let the assumed dimensions be as shown in Fig. 22.

Then $F = b_1h_1 - b_2h_2 = 78$ square inches.

$$I = \frac{b_1h_1^3 - b_2h_2^3}{12} = 3{,}000 \text{ bi-quadratic inches.}$$

$$e = \frac{h}{2} = 9''.$$

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$l =$ the reach of the punch $+ \frac{h}{2} = 27''$.

$P = 60{,}000$ lbs. as determined above.

Then $p = \frac{P}{F} = \frac{60{,}000}{78} = 770.$

$$p' = \frac{Ple}{I} = \frac{60{,}000 \times 27 \times 9}{3{,}000} = 4{,}860.$$

$p + p' = 5630 =$ the maximum fibre stress in the section. The average strength of cast iron such as is used for machinery castings, is about 20000 lbs, per square inch. The factor of safety against external stress in the case that has been assumed would be $= 20000 \div 5630 = 3.5$ and this is too small. There are two reasons why a large factor of safety should be used in this design. 1st. When the punch goes through the plate the yielding is sudden and a severe shock results, which has to be sustained by the frame which for other reasons is made of unresilient material. 2d. Since the frame is of cast iron there will necessarily be shrinkage stresses which will have to be sustained by the frame in addition to the external stresses, these shrinkage stresses cannot be calculated and therefore cannot be provided against except by a large factor of safety. Also since cast iron is strong to resist compression, and weak to resist tension, and since the maximum fibre stress is tension on the inner side, the metal can be more satisfactorily distributed than in the assumed section by being thickened where it is subjected to tension, as at A Fig. 23. If however, there is a very thick body of metal at (a) the result would be spongy metal and excessive shrinkage stresses and the form B would probably be preferable, the metal being arranged better for proper cooling and also for the resisting of flexure stress.

Let dimensions be now assigned to B and let the cross section be checked for strength as before. AA is a line through the centre of gravity of the section and it is found to be at a distance of $7''$ from the tension side. The required valves are as follows :

$e = 7''$
$l =$ reach of the punch $+ e = 18'' + 7'' = 25''$
$F = 158$ square inches.
$I = 6260$ bi-quadratic inches.
$P = 60000$ lbs.

$$\text{then } p = \frac{P}{F} = \frac{60,000}{158} = 380,$$

$$\text{and } p' = \frac{Ple}{I} = \frac{60,000 \times 25 \times 7}{6,260} = 1,680,$$

therefore $p + p' = 2060$ which would be the maximum fibre stress. The factor of safety would be $= 20000 \div 2060 = 10$ nearly. This section therefore fulfills the requirement of strength, and the material is well arranged for cooling with little shrinkage stress, and with no spongy spots.

Having now determinee the form and dimensions of the section AB, Fig. 20, it is next necessary to proportion the rest of the frame. All sections between DG and HK are subjected to the same stresses and should therefore have the same form and area. The cantilever DEF must resist a vertical shear $= P$, and also a flexure stress $= Px$; x being the distance from the section considered to the line of applition of P. The outline FEGB of the frame should now be drawn, such dimensions of cross section being assumed that the outline curve is smooth and of good appearance; then two or three sections between ED and P should be checked to insure that the factor of safety equals about 10 in all parts. The rib may be treated in either of two ways. 1st. It may be tapered as in (a) in which case the outline would be a parabolic curve; or 2d. It may be made of uniform thickness in the elevation (a) and tapered as shown in the plan (b) the proper lines in this case being straight lines. The necessary modifications of the frame to provide for its support, and for the constrainment of the actuating gear may be worked out as in Fig. 24. A is the pinion on the pulley shaft from which the power is received; B is the gear on the main shaft; C and D are parts of the frame that are added to supply bearings for the main shaft; E furnishes the guiding sur-

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faces for the punch "slide". The method of supporting the frame is as shown, the support being cut under at F for convenience to the workman. From this general conception of the machine the details may be worked out and so the design completed.

A slotting machine is selected for the second illustration of the design of machine frames. (See Fig. 25.) It is specified that the slotter shall cut at a certain distance from the edge of any piece, and the dimension AH is thus determined. The table G must be held at a convenient height above the floor, and PK must provide for the required range of "feed." K is cut under for convenience and carried to the floor line as shown. It is required to "slot" a piece of given vertical dimensions and the distance from the surface of the table to E is thus determined. To design the frame it is necessary to know the direction, intensity and point of application of the force to be resisted, *i. e.*, the reaction that comes upon the cutting tool. The tool A cuts on the down stroke and the resistance to the cut is transmitted through the "cutting bar" B, the connecting rod C and the crank disc Q, to the bearing N which is a part of the frame. A force equal to the resistance of the tool, acting vertically at the end of N needs to be resisted by the frame. Let the dimension LM be assumed so that it shall be in correct proportion to the necessary length and height of the machine. The curves LS and MT may be drawn for bounding lines of a box frame whose upper end will support the top of the guide, and whose form shall be a proper one to resist stress transmitted from N through the compression rib D. M should be carried to the floor line as shown and not cut under as before explained. E is a tension or compression rib that serves to transmit to the main frame the horizontal stresses that come on the tool. None of the part DNE, nor that which serves to support the cone and gears on the other side of the frame, should be made flush with the surface LSTM, because

nothing should interfere with the continuity of the curves LS and TM. *The supporting frame of a machine should be clearly outlined, and other parts should appear as attachments.* The member VW should be designed so that its inner outline is nearly parallel to the outline of the cone pulley and should be joined to the main frame by a curve. The outer outline should be such that the width of the member increases slightly from W to V, and should also be joined to the main frame by a curved outline. The amount of metal in the cross sections of the frame and its arrangement may be controlled by the core, and is dictated by the maximum stress. The method in this case as in that of the punch, is to assume a cross sectional area and arrangement and then to check for the maximum fibre stress.

The punch and slotting machine illustrate what may be called "open side machine framing." Another kind of framing which requires consideration may be represented by a triangle. (See Fig. 26.) The force acts between A and D tending to separate them or to bring them nearer together, and is resisted by the tension or compression members, AB and AC and the flexure member BC. Center crank engines, steam hammers, many kinds of presses, etc., are examples of this kind of framing. To illustrate, let it be required to design the standards and bed plate of an 8"x12" vertical engine with a center crank. Without discussing the character and intensity of the stress, it will be assumed that the greatest accidental stress that can come on the stress members, when water is carried over with the steam is $= 65000$ pounds, and is applied as shown at P. Fig. 27. The dimension AB is determined by the already worked out dimensions of the crank; the dimension CD, results from the cylinder design, and the lines AC and BD are thus determined, which are the inner bounding outlines of the standards to be designed. P resolved into components along the axes of the standards $= 32800$ pounds. Each standard is required to

safely resist a tensile or compressive stress of 32800 pounds. These standards will have to resist accidental flexure stresses, and these, as well as the tendency to buckle, are best resisted by the box form of cross section, and that form will therefore be selected. There is no reason for massing the metal in any part of the section, and it will be made of a uniform thickness of 3/8″ because a casting of this size cannot be cast thinner than that successfully. The external dimensions are determined as follows: If the dimension GH were made equal to the external diameter of the cylinder, the cylindrical outline would be obliterated, and this is not allowable because the internal form, indicating the purpose of a machine part, should be indicated if possible by the external form; a part, therefore, of the cylindrical outline should be allowed to appear in this engine cylinder, and the width GH is theforefore made equal to 8″, equal to the internal cylinder diameter. The dimension EF should be such that it appears properly proportioned to CD; in this case say 4″. These dimensions are shown in the cross section, Fig. 28. The unit stress that can come upon this section is equal to the total maximum stress, = 32800 pounds, divided by the area of the metal in the cross section, = 8.5″ square inches.

$$\text{Unit stress} = \frac{32800}{8.5} = 3850 \text{ pounds.}$$

The ultimate strength of the cast iron is 20000, and the factor of safety is therefore $= \frac{20000}{3850} = 5.2$. This is a factor of safety over the maximum stress that can come on the part and 2 might be considered a sufficiently large value, the factor of safety over the ordinary working stresses being so very much greater. The thickness of the metal in the cross section cannot be reduced, however, and the external dimensions could not be made smaller without interfering with the correct appearance; also a large factor of safety is desirable in this case as the unresilient material in the member is liable

[illegible]

to severe shocks. These members are supports as well as stress members, and therefore they need to be widened toward the base, as shown in both elevations, (Fig. 27,) in order to have correct appearance. These supports will be found amply strong to resist buckling if checked by Euler's formula. If this were a horizontal engine, these would be simple stress members, and should have parallel sides. See the frames of the Straight Line engines.

The bed plate may be considered as a beam built in at both ends, and with a central load of 65000 pounds. KL is the only section that needs to be checked, as it is the section of maximum moment, and is reduced in vertical dimensions, because of the necessary opening for the reception of the main boxes. The cross section on the line KL may be drawn, the moment of inertia and the distance from the neutral axis to the outer fibre in tension, may be calculated, and these values substituted in the moment formula solved for unit stress in the outer fibre, would give the value of the latter, which could be compared with the value of safe stress for the material. The thickness of ⅝″ for this bed plate gives ample strength. Note that excess of material is not required here to absorb vibrations, since the tendency to vibration in a vertical engine is in a vertical direction, and hence is absorbed by the foundation. The bottom flange affords a better bearing on the foundation, and is also effective to increase the moment of inertia of the cross section at KL, and hence to increase the strength of the bed to resist flexure.

8. On the design of Journals.

Journals, and the bearings or boxes with which they engage, are the kinematic elements that are used to constrain motion of rotation or vibration about axes in machines.

Journals are usually cylindrical in form, although they may be conical, or in rare cases, spherical.

As far as their design is concerned, journals may be divided into two classes: First, those in which the applied stresses dictate the design, and second, those in which maintenance of form is of chief importance. Thus the main journal of a steam engine shaft is designed from a consideration of the stresses to which it is subjected, while the main journal of the spindle of a universal grinding lathe is made of such material and size that it shall remain truly cylindrical for the longest possible time, the stresses being so small that they are not taken into the account. To still further illustrate the first class, let Fig. 29 represent a pulley on the end of an overhanging shaft, carrying a belt ABC. Rotation is in the direction indicated by the arrow, and the belt tensions are T_1 and T_2. The journal J is supported by a box, or bearing D, and is subjected to the following stresses: (a) Torsion, measured by the moment $(T_1 - T_2)$ r; (b) flexure, measured by the moment $(T_1 + T_2)$ a, (This assumes that the box is "self-adjusting"); (c) shear due to the force $T_1 + T_2$. This journal must therefore be so designed that it shall not be ruptured nor unduly strained by the torsional, transverse or shearing force. This same force $T_1 + T_2$ brings a pressure to bear between the rubbing surfaces, at right angles to the plane EF, and the design must also insure running without undue heating such as might result in the destruction of the lubricant, or of the surfaces of the journal and bearing. The pressure $T_1 + T_2$ might be sufficiently great so that the lubricant would be squeezed out from between the metal surfaces and they would come into contact, and heating and abrasion would result. In speaking of the area of a journal in what follows the projected area, *i. e.*, the length × diameter of the journal, will be meant.

The allowable pressure per square inch of area of a journal varies with several conditions. To make this clear let it be supposed that a small drop of oil be put in the middle of a small, accurately finished surface plate; suppose that another exactly similar plate be superimposed upon it for just an instant; the oil drop will be spread out because of the force due to the weight of the upper plate. If the plate be allowed to remain a longer time the oil will be still further spread out, and if its weight were sufficient the oil would finally be entirely squeezed out from between the plates and the metal surfaces would come in contact. It will be seen then that the squeezing out of the oil from between the rubbing surfaces of a journal and its box is a function of time as well as of pressure. If in addition to the pressure the surfaces be moved over each other, the removal of the oil is facilitated, and the greater the velocity of movement the more rapidly will the oil be removed, and therefore the squeezing out of the oil is also a function of the velocity of rubbing surface.

If a journal be subjected to continuous pressure in one direction, as for instance a shaft with a constant belt pull, or with a heavy fly wheel upon it, this pressure has sufficient time to act, and is therefore effective for the removal of the oil. But if the direction of the pressure be periodically reversed, as in the crank pin of a steam engine, the time of action is less, the tendency to remove the oil is reduced, and also the oil has opportunity to return between the surfaces. It will be clear then that a higher pressure per square inch of journal would be allowable in the second case than in the first.

If not only the direction of pressure, but also the direction of motion be reversed, as in the cross head pin of a steam engine, the oil not only has an opportunity to return between the surfaces, but is also assisted in doing so by the reversed motion. Therefore in this case a still higher pressure per square inch of journal is allowable. Practical experience bears out these conclusions. Thus in case of journals with

Some Strength of Materials Constants.	*Per ct. Carbon.*	*Stress at Elas. Lim. Tension.*	*Stress Ultimate Tension.*	*Stress at Elas. Lim. Comp'n.*	*Stress Ultimate Comp'n.*	*Stress Ultimate Shearing*	*Stress in Outer Fib. Flexure.*	*Modulus of Elasticity. Shearing.*	*Modulus of Elasticity. Tension.*
BESSEMER AND SIEMENS-MARTIN STEEL.	.15	42,000	63,000	39,000		48,000		9,000,000	30,000,000
	.20	47,000	68,000	43,000		53,000			
	.50	48,000	80,000	46,000		57,000			
	.70	53,000	89,000	53,000		60,000			
	.80	57,000	103,000	63,000		68,000			
	.96	69,000	118,000	71,000		83,000			
High Grade Wr'ght Iron		28,000	56,000	28,000		40,000	Elas. Lim. 52,000	9,000,000	28,000,000
Common Wrought Iron		22,000	40,000	22,000		32,000		9,000,000	28,000,000
Crucible or Tool Steel .		58,000	116,000	58,000				12,400,000	31,000,000
Malleable Cast Iron . .			35,000						
Steel Castings		29,000	47,000	29,000					19,000,000 to 31,000,000 Average, 25,000,000
Cast Steel			10,000 to 35,000. 20,000 av.		56,000 to 145,000. 90,000 av.	18,000 to 20,000	Ultimate 30,000 to 54,000. 42,000 av.	7,000,000	13,000,000

the direction of pressure constant, it is found that with ordinary conditions of lubrication the heating and "seizing" or "cutting" occur quickly if the pressure per square inch of journal exceed about 380 lbs. (See Mr. Tower's experiments in the Minutes of the Inst'n of Mech'l Engineers.) But in the crank pins of punching machines where the pressure acts for an instant, with quite an interval of rest, and where the velocity of rubbing surface is very low indeed, the pressure is often as high as from 2000 to 3000 lbs. per square inch, and there is no tendency to heating or abrasion. In engine crank pins the pressure may be from 400 to 800 lbs. depending on the velocity of rubbing surface, and in cross head pins where the velocity is always low it may be from 600 to 1000 lbs. The value that is to be used in each particular case must be decided by the judgment of the designer.

But even if the conditions are such that the lubricant is retained between the rubbing surfaces, yet heating may occur. There is always a frictional resistance at the surface of the journal; this resistance may be reduced, (a) by insuring accuracy of form and perfection of surface in the journal and its bearings; (b) by insuring that the journal and its bearing are in contact throughout their entire surface, by means of rigidity of framing or self-adjusting boxes, as the case may demand; (c) by selecting a lubricant that is suitable to meet the conditions, and maintaining the supply to the bearing surfaces. By these means the friction may be reduced to a very low value, but it cannot be reduced to zero.

There must be some frictional resistance, and it is always converting mechanical energy into heat, and it is this heat that raises the temperature of the journal and its bearing. If the heat thus generated is conducted and radiated away as fast as it is generated, the box remains at a constant low temperature. If, however, the heat is generated faster than it can be disposed of, the temperature of the box rises till its capacity to radiate heat is increased by the increased difference of temperature of the box and the surrounding

air, so that it is able to dispose of the heat as fast as it is generated. It will be easily understood that this temperature that is necessary to establish the equilibrium of heat generation and disposal, might, under certain conditions be high enough to destroy the lubricant, or even to melt out a babbitt metal box lining. Suppose now that a journal is running under certain conditions of pressure and surface velocity, and that it remains entirely cool. Suppose next that while all other conditions are kept exactly the same, the velocity is increased. All modern experiments on the friction in journals show that the friction increases with the increase of the velocity of rubbing surface. Therefore, the increase in velocity would increase the frictional resistance at the surface of the journal, and also the space through which this resistance acts would be greater in proportion to the increase in velocity. The work of the friction at the surface of the journal is therefore increased, because both of the factors of which it is composed, *i. e.*, the force and the space are increased. It is this work of friction which has been so increased that produces the heat that tends to raise the temperature of the journal and its box. The rate of generation of heat has therefore been increased by the increase in velocity, but the box has not been changed in any way and therefore its capacity for disposing of heat is the same as it was before, and hence it will be seen that the tendency of the journal and its bearing to heat is greater than it was before the increase in velocity. Some change in the proportions of the journal must be made in order to keep the tendency to heat the same as it was before the increase in velocity. If the diameter of the journal be increased, the radiating surface of the box will be proportionately increased. But the space factor of the friction will be increased in the same proportion, and therefore it will be apparent that this change has not affected the relation of the rate of generation of heat to the disposal of it. But if the length of the journal be increased the work of friction is the same as before and the radiating surface of the box is increased and the

tendency of the box to heat is reduced. If therefore the conditions are such that the tendency to heat in a journal, because of the work of the friction at its surface, is the vital point in design, it will be clear that the length of the journal is dictated by it, but not the diameter. The reason why high speed journals have greater length in proportion to their diameter than low speed journals will now be apparent.

For the illustration of the application of these principles to the practical designing let it be required to design a crank pin for an engine as follows:

Let D = the cylinder diameter = 12″.

" p = the mean effective pressure per square inch on the piston = 70 lbs.

Let N = the number of revolutions per minute = 200.

" f = the coefficient of friction.

" d = the diameter of the crank pin.

" l = the length of the crank pin.

The force of friction at the surface of the journal $= p D^2 \frac{\pi}{4} f$.

The space acted through per minute by the force of friction $= \pi d N$.

The area of the journal $= d l$.

The work of the friction at the surface of the journal per square inch of area of the journal =

$$\frac{p D^2 \pi f \times \pi d N}{4 d l} = \frac{p D^2 \pi^2 N f}{4 l} \qquad (1)$$

Experiments on a large number of marine engines show that the crank pins will run cool when the work of friction per square inch of area of the pin does not exceed a value equal 1,000,000f inch pounds, f being the coefficient of friction. It may be safely assumed that the pin of a stationary engine will run cool if this value be not exceeded. Therefore if this value be equated with equation (1) above and solved for l, the length of the crank pin will be obtained which in the given engine will limit the work of friction at the surface of the journal per square inch of area of the journal, to the value that has been shown to be entirely safe

The coefficient of friction cancels out because ...umed that its value in the crank to be designed ... cranks experimented upon is the same since the ... the surfaces, the methods of lubrication and the ...must be nearly identical.

$$\frac{D^2 \pi^2 N}{4 l} = 1{,}000{,}000.$$

$$\therefore l = \frac{p D^2 \pi^2 N}{4{,}000{,}000} = \frac{70 \times 12^2 \times \pi^2 \times 200}{4{,}000{,}000} = 4.96'', \text{ say } 5''$$

...meter of the crank pin may be determined as fol... greatest working pressure that can come on the ... taken as the product of the maximum boiler pres... to say 100 lbs. per square inch multiplied by the ... piston, and a safe working pressure per square inch ... the pin is say 500 lbs. ... the necessary area of the pin, ... multiplied by l, is therefore equal to the greatest ...ressure divided by 500 lbs.

$$\ldots d\,l = \frac{100 \times 12^2 \pi}{4} \div 500 = 11300 \div 500 = 22.6 \text{ sq.}$$

$$\text{And } d = \frac{22.6}{5} = 4.5''.$$

...ross head pin of the same engine the pressure is ... same, but the design is modified by variation in ... conditions. Thus the surface velocity here is so ... the work of friction is inconsiderable and does not ... design. The pin may therefore be made of any ...t length, and the important thing is to make the ...e pin such that the pressure per square inch of that ... not be able to squeeze out the lubricant. It has ... that a higher unit pressure is allowable in this ... pin than in the crank pin because of the low velocity, ...versal of pressure, and of motion. Suppose that in ...he allowable pressure is 700 per square inch of area of ... The maximum working pressure is 11300 as above; ... area of the pin would equal $11300 \div 700 = 16.2$ ...ches. If a length of 5″ be assumed then the diam...d equal $16.2 \div 5 = 3.24$, say $3\frac{1}{4}''$.

The cross head pin and the crank pin just designed may be checked for strength and stiffness. See Fig. 30. The cross head pin AB is supported at both ends, and the box C has a bearing throughout its entire length. The force P, applied as shown, will not tend to deflect the pin, because as soon as the least yielding occurs at the middle, the pressure will be concentrated at the ends, if the box is unyielding, and the pin must yield by shearing instead of by flexure. It will be evident without figures that there is not the slightest danger of this pin shearing by any stress that could possibly come upon it. If the crank be of the "centre crank type," the conditions would be exactly the same for the crank pin as for the cross head pin ; if it be of the "side crank" type, as shown in Fig. 31, the crank box bearing throughout the entire length of the pin, any tendency to yield on the part of the pin would be accompanied by the concentration of the pressure nearer to the support (unless the connecting rod should yield sideways and allow the box to adjust itself to the new position of the pin, which would be a fault of the rod and not of the pin), and the pin in this case, as in that of the cross head pin, would yield by shearing, and it is clearly safe against that danger. Since it is a cantilever with a uniform load, it might yield by the breaking of the cross section AB where the pin joins the crank. Checked by the formula for strength, it will be found amply strong.

The journals that are subjected to slight stresses, and that are required above all other things to maintain accuracy of form, must be designed from precedent or according to the judgment of the designer, as no theory can lead to correct proportions.

9. Thrust journals. When a rotating machine part is subjected to pressure parallel to the axis of rotation, means must be provided for the safe resistance of that pressure. In the case of vertical shafts the pressure is due to the weight of the shaft and its attached parts ; as the shafts of turbine water-wheels that rotate about vertical axes. In other cases the pressure is due to the working force ; as the shafts of propel-

or wheels, the spindles of a drilling lathe, etc. The end thrust of vertical shafts is very often resisted by simply squaring up the end of the shaft and inserting it in a bronze or brass bush which embraces the end of the shaft to prevent lateral motion, as in Fig. [illegible]. If the pressure be too great the end of the shaft may be enlarged so as to increase the bearing surface and thereby reduce the pressure per square inch. This enlargement must be within narrow limits, however. See Fig. [illegible]. AB is the axis of rotation and ACD is the rotating part, its bearing being enlarged at CD. Let the conditions of wear be considered. The velocity of rubbing surface varies from zero at the axis to a maximum at C and D. It has been seen that the increase of the velocity of rubbing surface increases both the force the friction and the space through which that force acts; it therefore increases the work of the friction and therefore the tendency to wear. From this it will be seen that the tendency to wear increases from the centre to the circumference of this "radial bearing," and that, after the bearing has run for awhile, the pressure will be localized near the centre and heating and abrasion may result. For this reason, where there is severe stress to be resisted, the bearing is usually divided up into several parts, the result being what is known as a "coller thrust bearing," as shown in Fig. 34. By the increase in the number of collers, the bearing surface may be increased without increasing the tendency to unequal wear. The radial dimension of the bearing is kept as small as is consistent with the other considerations of the design.

It is found that the "Tractrix," the curve of constant tangent, gives the same work of friction, and hence the same tendency to wear in the direction of the axis of rotation, for all parts of the wearing surface. (See Church's Mechanics, page 181). This is without doubt the best form for a thrust bearing, but the difficulties in the way of the accurate production of its curved outlines have interfered with its extensive use.

The pressure that is allowable per square inch of projected

area of the bearing surface varies in thrust bearings with several conditions, as it does in journals subjected to pressure at right angles to the axis. Thus in the pivots of turn-tables, swing bridges, cranes, and the like, the movement is slow and never continuous, often being reversed, and also the conditions are such that "bath lubrication" may be used, and the allowable unit pressure is very high; equal often to 1500 pounds per square inch, and in some cases greatly exceeding that value. The following table may be used as an approximate guide in the designing of thrust bearings.

The material of the thrust journal is wrought iron or steel, and of the bearing is of bronze or brass (babbitt metal is seldom used for this purpose). Bath lubrication is used, *i. e.*, the running surfaces are submerged constantly in a bath of oil.

Mean velocity of rubbing surface. Ft. per min.	Allowable unit pressure. Lbs. per square inch of projected area of the rubbing surface.
Up to 50	1000
50 to 100	600
100 to 150	350
150 to 200	100
Above 200	50

If the journal is of cast iron and runs on bronze or brass, the values of allowable pressure given should be divided by 2.

Examples to illustrate the design of thrust journals. 1st. It is required to design a thrust journal whose outline is a "tractrix," and which is required to support a vertical shaft which, with its attached parts, weighs 2,000 lbs., and runs at a rotative speed of 200 revolutions per minute. The dimensions of the thrust journal are as yet unknown and therefore the velocity of rubbing surface must be estimated. Suppose that the mean diameter of the journal is 2″, then the mean velocity of rubbing surface will be $2 \times \pi \times N \div 12 =$ 103 feet per minute. This is so near the limit in the table between an allowable pressure of 350 and 600 that an intermediate value may be used, say 450 pounds. The projected area of the journal then will equal the total pressure divided by the allowable pressure per square inch of the journal = $2{,}000 \div 450 = 4.44''$ square inches. The journal must not

[redacted]

[redacted]

be pointed as in (a) Fig. 35, but must be as shown in (b). The dimension BC may be assumed equal to 1″. The projected area of the journal is equal to the circular area whose diameter is AD minus the circular area whose diameter is BC, and this may be equated with the required value equal 4.44 and the equation solved for the required dimension AD.

Thus $\frac{\overline{AD}^2 \pi}{4} - \frac{\overline{BC}^2 \pi}{4} = 4.44$

Therefore $\overline{AD}^2 = \frac{4.44 \times 4}{\pi} + \overline{BC}^2$

$AD = \sqrt{6.68} = 2.58''$.

In order now to draw the required journal, lay off from the axis EF the distance EG equal half AD, and through the point G draw a "tractrix" whose constant tangent is equal to EG, continuing the curve till it reaches a point C, such that FC is equal to half the assumed value of BC. The vertical dimension of the journal is thereby determined and the corresponding curve BH may be drawn on the other side of the axis EF.

2d Example. It is required to design the collar thrust journal that is to receive the propelling pressure from the screw of a small yacht. The necessary data are as follows:

The maximum power delivered to the shaft is 70 H. P.

The pitch of the screw is 4 feet.

The slip of the screw is 20%.

The shaft revolves 250 times per minute.

The diameter of the shaft is 4″.

For every revolution of the screw the yacht moves forward a distance = 4 ft. less 20% = 3.2 ft., and the speed of the yacht in feet per minute = 250 × 3.2 = 800.

70 H. P. = 70 × 33000 = 2310000 foot pounds per minute. This work may be resolved into its factors of force and space, and the propelling force is equal to 2310000 ÷ 800 = 2900 lbs. nearly.

The shaft is 4″ diameter, and the collars must project beyond its surface Estimate that the mean radius of the rub-

bing surface is 4.5″, then the mean velocity of rubbing surface would equal $4.5 \times \pi \div 12 \times 250 = 294$ feet per minute. The allowable value of pressure per square inch of journal surface for a velocity above 200 ft. per minute is 50 lbs. The necessary area of the journal surface is therefore $= 2900 \div 50 = 58$ square inches. It has been seen that it is desirable to keep the radial dimension of the collar surface as small as possible in order to have as nearly the same velocity at all parts of the rubbing surface as possible. The width of collar in this case will be assumed $= .75''$, then the bearing surface in each collar

$$= \frac{5.5^2 \times \pi}{4} - \frac{4^2 \times \pi}{4} = 23.7 - 12.5 = 11.2$$

Then the number of collars equal the total required area divided by the area of each collar $= 58 \div 11.2 = 5.18$ say 6.

10. Bearings and boxes. The function of a bearing or box is to insure that the journal with which it engages shall have an accurate motion of rotation or vibration about the given axis. It must therefore fit the journal without lost motion; must afford means of taking up the lost motion that results necessarily from wear; must resist the stresses that come upon it through the journal without undue yielding; must have the wearing surface of such material as will run in contact with the material of the journal with the least possible friction, and least tendency to heating and abrasion, and must usually include some device for the maintenance of the lubrication. The selection of the materials and the providing of sufficient strength and stiffness depends upon principles already considered, and so it remains to discuss the means for the taking up of necessary wear and for providing lubrication.

Boxes are sometimes made solid rings or shells, the journal being inserted endwise. In this case the wear can only be taken up by making the engaging surfaces of the box and journal conical, and providing for small permanent endwise movement either of the box itself, or of the part carrying the

journal. Thus, in Fig. 36, the collars for the preventing of end motion while running are jamb nuts, and by means of them the position of the journal relatively to the box B may be changed, and looseness between the journal and box may be taken up by moving the journal axially toward the left.

By far the greater number of boxes, however, are made in sections, and the lost motion is taken up by moving one or more sections toward the axis of rotation. The tendency to wear is usually in one particular direction, and in such a cast it serves to divide the box into halves. Thus, in Fig. 37, the journal rotates about the axis O, and all the wear is due to the pressure P acting in the direction shown, and the wear will all be at the bottom of the box, and it will suffice for the taking up of wear to dress off the surfaces at *a a*, and thus the box cap may be drawn further down by the bolts, and the lost motion is reduced to an admissible value. "Liners" or "shims," which are thin pieces of sheet metal, may be inserted between the surfaces of division of the box at *a a*, and may be removed successively for the lowering of the box cap as the wear renders it necessary. If the axis of the journal must be kept in a constant position, the lower half of the box must be capable of being raised.

Sometimes, as in the case of the box for the main journal of a steam engine shaft, the wear is in several directions. Thus, in Fig. 38, A represents the main shaft of an engine, and there is a tendency to wear in the direction B because of the weight of the shaft and its attached parts; there is also a tendency to wear because of the pressure that comes through the connecting rod and crank. The direction of this pressure is constantly varying, but the average direction on forward and return stroke may be represented by C and D. Provision needs to be made for taking up wear in these three directions. If the box be divided on the line E F it will be seen that this wear will be approximately taken up. Usually, however, in the larger engines the box is divided into four sections, A, B, C and D, (Fig. 39,) and A and C are capable of being moved toward the shaft by means of screws or

wedges, while D may be raised by the insertion of "shims."

The lost motion between a journal and its box is sometimes taken up by making the box as shown in Fig. 41. The external surface of the box is made conical and fits in a conical hole in the machine frame. The box is split entirely through at A, parallel to the axis, and partly through at B and C The ends of the box are threaded and the nuts E and F are screwed on. After the journal has run long enough so that there is an unallowable amount of lost motion, the nut F is loosened and E is screwed up; the effect being to draw the conical box further into the conical hole in the machine frame, and the hole through the box is thereby closed up, and the lost motion is reduced. After this operation the hole cannot be truly cylindrical, and so, if the cylindrical form of the journal has been maintained, it will not have a bearing throughout its entire surface. This is not usually of very great importance, however, and the form of box has the advantage that it holds the axis of the journal in a constant position.

All boxes in self contained machines like engines or machine tools, need to be rigidly supported to prevent the localization of pressure, since the parts that carry the journals are made as rigid as possible. In line shafts and other parts carrying journals, when the length is great in comparison to the lateral dimensions, some yielding must necessarily occur, and if the boxes were rigid, localization of pressure would result, and so "self-adjusting boxes" are used. A point in the axis of rotation that is the centre of the length of the box is held immovable, but the box is free to move in any way about this point, and thus adjusts itself to any yielding of the shaft. This result is attained as shown in Fig. 40. O is the centre of the motion of the box; B and A are spherical surfaces on the box whose centre is O, and the support for the box carries internal spherical surfaces which engage with A and B, and so the point O is always held in a constant position but the box itself is free to move in any way about O as a centre. Therefore the box adjusts itself, within limits,

to any position of the shaft, and so the localization of pressure is impossible.

In thrust bearings for vertical shafts, the weight of the shaft and its attached parts serves to hold the rubbing surfaces in contact and so the lost motion is taken up by the shaft following down as the wear occurs. In collar thrust bearings for horizontal shafts the design is such that the bearing for each collar is separate and adjustable, and so the pressure on the different collars may be equalized. (For complete and varied details of marine thrust bearings see "Maw's Modern Practice in Marine Engineering.")

Lubrication of journals. The best method of lubrication is that in which the rubbing surfaces are constantly submerged in a bath of lubricating fluid. This method should be employed whenever possible, if the pressure and surface velocity are high. Unfortunately it cannot be used in the majority of cases. Let J, Fig. 42, represent a journal with its box, and let A, B and C be oil holes. If oil be introduced into the hole A, it will tend to flow out from between the rubbing surfaces by the shortest way; or, in other words, it will all come out at D. A small amount will probably go toward the other end of the box because of capillary attraction, but in most cases none of it will get as far as the middle of the box. Also if oil be introduced at C it will come out at E. A constant feed of oil therefore might be maintained at A and C and yet the middle of the box might run dry. If the oil be introduced at B, however, it tends equally to flow in both directions, and the entire journal is lubricated. From this follows the conclusion that oil ought when possible, to be introduced at the middle of the length of a cylindrical journal. If a conical journal runs at a high velocity, the oil under the influence of centrifugal force tends to go to the large end of the cone, and therefore the oil should be introduced at the small end and its distribution over the entire journal surface will be insured.

If the end of a vertical thrust journal, whose outline is a cone or a tractrix as in Fig. 43, dips into a bath of oil B, the

oil will be carried by its centrifugal force if the velocity be high, up between the rubbing surfaces, and will be delivered into the groove A A. If holes connect A and B, gravity will return the oil to B, and so a constant circulation will be maintained. If the thrust journal has simply a flat end, as in Fig. 44, the oil should be supplied at the centre of the bearing, centrifugal force will then distribute it over the entire surface. Vertical shaft thrust journals may usually be arranged to run in an oil bath. Marine collar thrust journals are always arranged to run in an oil bath.

Sometimes a journal is stationary and the box rotates about it, as in the case of a loose pulley, Fig. 45. If the oil be introduced into a tube A, as is often done, its centrifugal force will carry it away from the rubbing surface. But if a hole be drilled in the axis of the journal, the lubricant introduced into it will be carried to the rubbing surfaces as required.

If a journal is carried in a rotating part at a considerable distance from the axis of rotation, and it requires to be oiled while in motion, a channel may be provided from the axis of rotation where oil may be introduced conveniently, to the rubbing surfaces, and the oil will be carried out by centrifugal force. Thus Fig. 46 shows an engine crank in section. Oil is introduced at O, and centrifugal force carries it through the channel provided to *a*, where it serves to lubricate the rubbing surfaces of the crank pin and its box.

If a journal is carried in a reciprocating machine part, and requires to be oiled while in motion, the "wick and wiper" method is one of the best. See Fig. 47. An ordinary oil-cup with an adjustable feed is mounted in a proper position opposite the end of the stroke of the reciprocating part, and a piece of flat wick projects from its delivery tube. A drop of oil runs down and hangs suspended at its end. Another oil-cup is attached to the reciprocating part, which carries a hooked "wiper" B, and whose delivery tube leads to the rubbing surfaces to be lubricated. When the reciprocating part reaches the end of its stroke the wiper picks off the drop

of oil from the wick, and it runs down into the oil-cup C, and thence to the surfaces that are required to be lubricated. It will be seen that this plan applies to the oiling of the cross head pin of a steam engine. The same plan is also sometimes applied to the crank pin, but here, through at least a part of the revolution, the tendency of the centrifugal force is to force the oil out of the cup, and therefore the plan of oiling from the axis is probably preferable.

When journals are lubricated by feed oilers, and are so located that they may escape attracting attention if the lubrication should fail for any reason, "tallow boxes" are used. These are cup like depressions that are usually cast in the box cap, and they communicate, by means of an oil hole, with the rubbing surface. These cups are filled with grease that is solid at the ordinary temperature of the box, but if there is the least rise of temperature because of the failure of the oil supply, the grease melts and runs to the rubbing surfaces, and so supplies the lubrication temporarily. This safety device is used very commonly on line shaft journals.

The most common forms of feed oilers are, 1st, the oil-cup with an adjustable valve that controls the rate of flow. 2d. The oil-cup with a wick feed (see Fig. 48.) The delivery has a tube inserted in it which projects nearly to the top of the cup. In this tube a piece of wicking is inserted, and its end dips into the oil in the cup. The wick, by capillary attraction, carries the oil slowly and continuously over through the tube to the rubbing surfaces. 3d. The cup with a copper rod. See Fig. 49. The oil-cup is filled with grease that melts with a very slight elevation of temperature, and A is a small copper rod that is dropped into the delivery tube and rests on the surface of the journal. The slight friction between the rod and the journal warms the rod and it melts the grease that is in contact with it, which runs down the rod to the rubbing surface. 4th. Sometimes a part of the surface of the bottom half of the box is cut away and a felt pad is inserted, its bottom being in contact with an oil bath. This

pad rubs against the surface of the journal, and it is kept constantly soaked with oil, and so maintains the lubrication.

11. Sliding surfaces. So much of the accuracy of action of machines depends on the sliding surfaces that their design deserves the most careful attention. The perfection of the cross section outline of the cylindrical or conical forms that are produced in the lathe, depends on the perfection of form of the spindle. But the perfection of the outlines of a section through the axis depends on the accuracy of the sliding surfaces. All of the surfaces produced by planers, and most of those produced by milling machines, are dependent for accuracy on the sliding surfaces.

Suppose that the short block A, Fig. 50, is the slider of a slider crank chain, and that it slides on a comparatively long guide D. The direction of rotation of the crank *a* is as indicated by the arrow. B and C are the extreme positions of the slider. The pressure between the slider and the guide is greatest at the mid position A, and at the extreme positions B and C it is only that pressure due to the weight of the slider. Also the velocity is a maximum when the slider is in its mid position, and decreases toward the ends, becoming zero when the crank *a* is on its centre. The work of friction is therefore greatest at the middle, and is very small near the ends. Therefore the wear would be greatest at the middle, and the guide would wear concave. If now the accuracy of a machine's working depends on the perfection of A's rectilinear motion, it will be seen that that accuracy will be destroyed as the guide D wears. If a gib, EFG, be attached to A, Fig. 51, and engage with D as shown to prevent vertical looseness between A and D, then if this gib be taken up to compensate wear after it has occurred, it will be seen that it will be loose in the middle position when it is tight at the ends, because of the unequal wear, and so it will fail of its function.

Suppose that A and D are made of equal length, as in Fig. 52. Then when A is in the mid position corresponding to maximum pressure and velocity and maximum wear, it is in

contact with D throughout its entire surface, and the wear is therefore the same in all parts of that surface. The slider retains its accuracy of rectilinear motion regardless of the amount of the wear, and the gib may be set up to compensate wear, and will be equally tight in all positions.

If A and B, Fig. 53, are the extreme positions of a slider, D being the guide, it will be seen that a shoulder would be finally worn at C, and so it would be better to cut away the material of the guide, as shown by the dotted line. Slides should always "wipe over" the ends of the guide when it is possible. Sometimes it is necessary to vary the length of stroke of a slider, and also to change its position relatively to the guide. Examples: "cutting bars" of slotting and shaping machines. In some of these positions therefore there will be a tendency to wear shoulders in the guide and also in the cutter bar itself. This difficulty is overcome if the slide and guide are made of equal length, and the design is such that when it is necessary to change the position of the cutter bar that is attached to the slide, the position of the guide may be also changed so that the relative position of slide and guide remains the same. The slide surface will then just completely cover the surface of the guide in the mid position, and the slide will wipe over each end of the guide, no matter what the length of the stroke may be.

In many cases it is impossible to make the slide and guide of equal length. Thus a lathe carriage cannot be as long as the bed; a planer table cannot be as long as the planer bed, nor a planer saddle as long as the cross head. When these conditions exist especial care should be given to the following: 1st The bearing surface should be made so large in proportion to the pressure to be sustained that the maintenance of lubrication shall be insured under all conditions. 2d. The parts that carry the wearing surfaces should be made so rigid that there shall be no possibility of the localization of pressure because of yielding and deformation.

As to their form, guides may be divided into two classes: angular guides and flat guides. Fig. 54 (a) shows an angu-

lar guide, the pressure being applied as shown. The advantage of this form is that as the rubbing surfaces wear the slide follows down and takes up the vertical wear, and also that in a lateral directiou. The objection to this form is that the pressure is not applied at right angles to the wearing surfaces, as it is in the flat guide shown in (b). Here however a gib, A, has to be provided to take up the lateral wear. The gib is either a wedge or else is backed up by screws. These forms of guides are used for planer tables. The weight of the table itself is depended on to hold the guide surfaces in contact, and if the table is light the tendency of a heavy side cut would be to force the table up one of the angular surfaces away from the other. If the table is very heavy, however, there is little danger of this, and so it will be clear why the angle guides of large planers are much flatter than those of smaller ones. In some cases one of the guides of a planer table is angular and the other is flat. The side bearings of the flat guide may then be omitted as the lateral wear is taken up by the angular guide. This arrangement is undoubtedly good if both guides wear down equally fast.

Fig. 55 shows three forms of guide such as are used for the cross slide of lathes, the vertical slide of shapers, the table slide of milling machines, etc. A is a taper gib that is forced in by a screw at D to take up the wear. When it is necessary to take up wear at B, the screw may be loosened and a shim or liner may be inserted between the surfaces at a. C is a thin gib, and the wear is taken up by forcing it against the sliding surface by means of several screws like the one shown. This form is not so satisfactory as the wedge gib, as the bearing is chiefly under the points of the screws, the gib being thin and yielding, whereas in the wedge there is complete contact between the metallic surfaces.

The sliding surfaces thus far considered have to be designed so that there will be no lost motion while they are moving, because they are required to move while the machine is in operation. The gibs have to be carefully de-

signed and accurately set so that the moving part shall be just "tight and loose," *i. e.*, so that it shall be free to move, but so that there shall not be the slightest lost motion, since that would interfere with the accurate action of the machine. There is, however, another class of sliding parts like the sliding head of a drill press, or the tail stock of a lathe, that are never required to move while the machine is in operation. It is only required that they shall be capable of being fastened accurately in a required position, their movement being simply to readjust them to other conditions of work, while the machine is at rest. No gib is necessary and no accuracy of *motion* is required. It is simply necessary to insure that their position is accurate when they are clamped for the special work to be done.

12. *Rivets and Riveted Joints.*—A rivet is a fastening used to unite metal plates or rolled structural forms, as in boilers, tanks, hulls of ships, built up trusses, etc. It consists of a head A, (Fig. 56) a straight shank B, and is inserted, usually red-hot, into holes either drilled or punched in the parts to be connected, and the projecting end of the shank is then formed into a head (see dotted lines) either by hand or machine riveting. A rivet is a permanent fastening and can only be removed by cutting off the head. A row of rivets joining two members is called a *riveted joint* or *seam of rivets*. In hand riveting, the projecting end of the shank is struck a quick succession of blows with hand hammers and formed into a head by the skill of the workman. An attendant holds a sledge or "dolly bar" against the head of the rivet. In "button set" or "snap" riveting, the rivet is struck a few heavy blows with a sledge to "upset" it, and then a die or "button set" (Fig. 57) is held with the spherical depression B upon the rivet and the head A is struck with the sledge, and the rivet head thus formed. In machine riveting a die similar to B is held firmly in the machine and another similar die opposite to it is attached to the piston of either a steam, hydraulic or pneumatic cylinder. If a rivet properly

placed in holes in the members to be connected, be put between the dies and pressure in the cylinder be raised, the movable die will be forced forward and a head formed on the rivet.

The relative merits of machine and hand riveting have been much discussed, and great advantages have been claimed for each. As a matter of fact either method if it be *carefully carried out* will produce a good serviceable joint. If in *hand* riveting the first few blows be light, the rivet will not be upset and the shank will be loose in the hole, and a leaky rivet results. If in machine riveting the axis of the rivet be not placed coincident with the axis of the dies, an off-set head results. (See Fig. 58.) In large shops where work must be turned out economically in large quantities, machines must be used. But there are always places inaccessible to machines where the rivets must be driven by hand.

Holes for the reception of rivets are usually punched, although for thick plates and very careful work they may be sometimes drilled. If a row of holes be punched in a piece of plate, and a similar row as to size and spacing be drilled in a piece of the same plate, testing to rupture will show that the punched plate is weaker than the drilled one. If the punched plate had been annealed it would have been nearly restored to the strength of the drilled one. Also if the holes had been punched $\frac{1}{8}''$ small in diameter and then reamed to size, the piece would be as strong as the drilled one. These facts, that have been experimentally determined, point to the following conclusions: First, punching injures the material and produces weakness. Second, the injury is due to stresses caused by the severe action of the punch since annealing, which furnishes opportunity for equalization of stress, restores the strength. Third, the injury is only in the immediate vicinity of the punched hole, since reaming out $\frac{1}{16}''$ on a side removes all the injured material. In ordinary boiler work the plates are simply punched and riveted. If better work is required the plates must be

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drilled, or punched small and reamed, or annealed. Drilling is slow and therefore expensive ; and annealing is apt to change the plates and also requires large expensive furnaces. Punching small and reaming is, therefore, the best method.

Riveted Joints are of two general kinds : First, *Lap Joints* in which the sheets to be joined are lapped upon each other and joined by a seam of rivets, as in Fig. 59 a. Second, *Butt Joints* in which the edges of the sheets abut against each other, and a strip called a "cover plate" or "butt strap" is riveted to both edges of the sheet as in c.

There are two kinds of riveting : Single, in which there is but one row of rivets, as in a, and double, where there are two rows. Double riveting is subdivided into "chain riveting," b, and "zig-zag" or "staggered" riveting, d.

Lap joints may be single, double chain, or double staggered riveted.

Butt joints may have a single strap as in c, or double strap, *i. e.*, an exactly similar one on the other side of the joint. Butt joints with either single or double strap may be single, double chain, or double staggered riveted.

To sum up, then, there are :

Lap Joints		Single	Riveted,
		Double Chain	"
		" Staggered	"
Butt Joints	Single Strap	Single	Riveted
		Double Chain	"
		" Staggered	"
	Double "	Single	Riveted
		Double Chain	"
		" Staggered	"

A riveted joint may yield in four ways. First, by the rivet shearing. Second, by the plate yielding to tension on the line AB, Fig. 60 a. Third, by the rivet tearing out through the margin as in c. Fourth, the rivet and sheet have a bearing upon each other at D and E (d) and are both in compression. If the unit stress upon these surfaces becomes too great the rivet is weakened to resist shearing, or the plate to

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resist tension and failure may occur. This pressure of the rivet on the sheet is called "bearing pressure."

As a preliminary to the designing of joints it is necessary to know the strength of the rivets to resist shear; of the plate to resist tension; of the rivets and plate to resist bearing pressure. These values must not be taken from tables of the strength of the materials of which the plate and rivets are made, but must be derived from experiments upon actual riveted joints tested to rupture. The reason for this is that the conditions of stress are modified somewhat in the joint. For instance, in single strap butt joints, and in lap joints, the line of stress being the centre line of plates, and the plates joined being offset, a flexure stress is introduced and the plate is weaker to resist tension; also if the joint yield to this stress in the slightest degree the "bearing pressure" is localized and becomes more destructive. Extensive and accurate experiments have been made upon actual joints and the results are available in Stoney's "Strength and Proportions of Riveted joints." The constants given are taken from this book.

	Iron.	*Steel.*
Ultimate shearing strength of rivets, single shear, . . .	40000	50000
" " " " " double " . . .	35000	44000
Ultimate tensile strength of plate between rivet holes, single shear, ...	40000	60000
Ultimate bearing pressure per sq. in. of diametral plane of rivet, single shear, ...	67000	95000
Ultimate bearing pressure per sq. in. of diametral plane of rivet, double shear, ...	89000	100000

The theoretical diameter of rivet for a given thickness of plate may now be determined.

Let d = diameter of the rivet hole.
" t = thickness of the plate.
" p = pitch of the rivets.
" T = ult. tensile strength of plate between rivet holes.
" S = " shearing " " rivets.
" C = " bearing pressure.

The strength of the rivet to resist shearing at AB, Fig. 61, should be equal to its strength to resist bearing pressure at AC and so the expressions for those strengths may be equated thus,

$$C\,t\,d = S\,d^2\frac{\pi}{4}$$

$$\text{Solving } d = \frac{C\,t}{S \times .7854} = \frac{67000}{40000 \times .7854}\,t = 2\,t.$$

From which it is seen that for equal strength to resist bearing pressure and shear, the diameter of the rivet should equal twice the thickness of the plate. Let the results thus derived be compared with the values that are used in actual practice. (See table.)

Comparative Values of Rivet Diameter for different Values of Thickness of Plate.

t	2t	$1.2\sqrt{t}$	d
3/16"	3/8	—	3/8
1/4"	1/2	9/16	1/2
5/16"	5/8	11/16	5/8
3/8"	3/4	3/4	5/8-3/4
1/2"	1	7/8	3/4-7/8
5/8"	1 1/4	15/16	3/4-1
3/4"	1 1/2	1 1/16	1-1 1/8
7/8"	1 3/4	1 1/8	1-1 3/8
1"	2	1 3/16	1-1 1/4
1 1/8"	2 1/4	—	1 1/8-1 3/8

The first column gives the thickness of the plate; the second the diameter of the rivet $= 2\,t$; the third gives the rivet diameter calculated from the formula of Professor Unwin,

[redacted]

$d = 1.2\sqrt{t}$; the fourth column gives rivet diameters as found in practice, taken from Stoney's book, page 12. It will be seen that $d = 2t$ agrees with practice up to ⅜ plate thickness, but for thicker plate gives values that are too large. The reason for this is that the difficulty in driving rivets increases very rapidly with their size; 1¼″ or 1⅜″ being the largest rivet that can be driven conveniently. The equality of strength to resist bearing pressure and shear is therefore sacrificed to convenience in manipulation.

As the diameter of the rivet is increased the area to resist bearing pressure increases less rapidly than the area to resist shear (the thickness of the plate remaining the same), the former varying as d, and the latter as d^2, therefore if d be not increased as much as is necessary for equality of strength, the excess of strength will be to resist bearing pressure. If the other parts of the joint be made as strong as the rivet in shear and this strength be calculated from the stress to be resisted, the joint will evidently be correctly proportioned.

To calculate the diameter of rivet for a butt joint with double cover plates. The rivet is in double shear and therefore ultimate bearing pressure = 89000 lbs. per square inch = C. And also ultimate shear pressure = 35000 lbs. per square inch = S′.

Equating as before $C\,d\,t = \frac{S'\,\pi\,d^2\,2}{4} = \frac{S'\,\pi d^2}{2}$.

From which $d = \frac{2\,C\,t}{S'\,\pi} = \frac{2 \times 89000 \times t}{\pi \times 35000} = 1.6\,t$ nearly.

Comparing results of this formula with tables of dimensions of practice, they will be found to be too large. The following empirical formulas may be trusted:

For thin plates, for iron $d = 1.3\,t$; for steel, $d = 1.25\,t$.
" thick " " " $d = 1.1\,t$; " " $d = 1.125\,t$.

The next value to be determined is the *pitch* of the rivets, *i. e.*, the distance from the centre of one rivet to the centre of the next one. (See Fig. 62.) It is required to make the pitch of such a value that the strength of the plate between

rivet holes to resist tension shall equal the strength of the rivet to resist shear. (It has already been seen that the strength to resist bearing pressure is equal to or greater than the strength to resist shear.) Equating expressions for shearing strength of the rivet, and tensile strength of the plate on a section through the rivet holes, and solving for p = *pitch*. For a single riveted lap joint,

$$\frac{\pi d^2}{4} S = T t (p - d).$$

$$\text{From which } p = \frac{.7854 d^2 S + T t d}{T t.}$$

Let $S = 40000$ and $T = 40000$.
Then if $t = \frac{1}{4}''$; $d = \frac{1}{2}''$; $p = 1.28''$.
Then if $t = \frac{3}{8}''$; $d = \frac{3}{4}''$; $p = 1.79''$.
Then if $t = \frac{1}{2}''$; $d = \frac{7}{8}''$; $p = 2.06''$.
Then if $t = 1''$; $d = 1\frac{1}{8}''$; $p = 2.12''$.

All of these agree with Stoney's Table of Boilermaker's Proportions, lap joints, iron plates and rivets, except for $t = \frac{1}{4}''$. This formula may, therefore, be trusted except for very thin plates.

To figure p for butt joints with double straps, single riveted. Since the rivet is in double shear,

$$p = \frac{2[.7854 d^2 S'] + T t d}{T t}; \quad S' = 35000 \text{ lbs. per sq. in. the}$$

value for double shear.

In case of steel plate and steel rivets, the values of the constants T and S need to be changed in above formulas. See values given above.

To figure the pitch of double riveted joints the method is the same. There are, however, two rivets now to support the strip of plate between holes instead of one, as in the single joint. (See Fig. 62.) Therefore the first formula for p becomes, simply multiplying the shearing strength by 2.

$$p = \frac{1.57 d^2 S + T t d}{T t}$$

For double shear $p = \frac{3.14 d^2 S' + T t d}{T t}$; S' being value for double shear.

The *margin* in a riveted joint is the distance from the edge of the sheet to the rivet hole. This must be made of such value that there shall be safety against failure by the rivet tearing out. There can be no satisfactory theoretical determination of this value; but practice and experiments with actual joints, show that a joint will not yield in this way if the margin be made $= d =$ diameter of the rivet. The distance between the centre lines of the rows in double chain riveting may be taken $= 2.5\,d$; and in double staggered riveting may be taken $= 1.88\,d$.

Thus the total width of lap for single riveting equals $3\,d$; in double chain riveting $= 5\frac{1}{2}d$ and in double staggered riveting $4.88\,d$.

It will be seen that a riveted joint cannot be as strong as the unperforated plates that it joins. The ratio of strength of joint to strength of plate is called *joint efficiency*. If the joint be equally strong to resist rupture in all possible ways, the joint efficiency would equal the ratio of area of plate through rivet section, to the area of unperforated section. Results obtained in this way differ somewhat from the results of actual tests and so the latter values should be used. See following tables:

Relative Efficiency of Iron Joints.	*Efficiency. Per cent.*
Original Solid Plate	100
Lap Joint, single riveted, punched	45
" " " drilled	50
" double "	60
Butt Joint, single cover, single riveted	45-50
" " " double "	60
" double " single "	55
" " " double "	66

[redacted]

Relative Efficiency of Steel Joints.	*Efficiency. Per cent.*		
Thickness of Plates	¼-⅜	½-⅝	¾-⅞
Original Solid Plate	100	100	100
Lap Joint, single riveted, punched	50	45	40
" " " drilled	55	50	45
" double " punched	75	70	65
" " " drilled	80	75	70
Butt Joint, double cover, double riveted, punched	75	70	65
" " " " " drilled	80	75	70

These tables are from Stoney's Strength and Proportions of Riveted Joints.

The following problem will serve to illustrate the design of riveted joints for boilers. It is required to design a horizontal tubular boiler 48″ diameter to carry a working pressure of 100 lbs. per square inch.

A boiler of this type consists of a cylindrical shell of wrought iron or steel plates made up in length of two or more courses or sections. Each course is made by rolling a flat sheet into a hollow cylinder and joining its edges by means of a riveted joint called the longitudinal joint or seam. The courses are joined to each other also by riveted joints called circular joints or seams. Circular heads of the same material have a flange turned all around their circumference by means of which they are riveted to the shell. See Fig. 63.

The proper thickness of plate may now be determined from, 1st. The diameter of shell $=48''$; 2d. The working steam pressure per square inch $=100$ lbs.; 3d. The tensile strength of the material used; let steel plates be used of 60000 lbs. specified tensile strength. As a preliminary let us investigate the conditions of stress upon the cross section of material cut by a plane. 1st. Through the axis, 2d, at right angles to the axis, of a thin hollow cylinder; the stress being due to the excess of internal pressure per square inch over the external pressure per square inch.

Let $l =$ the length of the cylindrical shell in inches.

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Let $d =$ the diameter of the cylindrical shell in inches.

Let $p =$ the excess of internal over external pressure per square inch.

Let $p_1 =$ unit stress in a longitudinal section of material of the shell due to p.

Let $p_2 =$ unit stress in a circular section of material of the shell due to p.

Let $t =$ thickness of plate.

Let $T =$ ultimate tensile strength of the plate.

In a longitudinal section the stress $= l\,d\,p$, and the area of metal sustaining it $= 2\,l\,t$. Then $p_1 = \frac{d\,p}{2\,t}$.

In a circular section the stress $= \frac{\pi\,d^2\,p}{4}$ and the area $= \pi d t$ (nearly). Then $p_2 = \frac{\pi\,d^2\,p}{4} \times \frac{1}{\pi\,d\,t} = \frac{d\,p}{4\,t}$.

Therefore the stress in the first case is twice as great as in the second; and a thin hollow cylinder is twice as strong to resist rupture on a circular section as on a longitudinal one. The latter only therefore need be considered in determining the thickness of plate.

Equating the stress due to p in a longitudinal section and the strength of the cross section of plate that sustains it, we have $l\,d\,p = 2\,l\,t\,T$

Therefore $t = \frac{d\,p}{2\,T} =$ the thickness of plate that would just yield to the unit pressure p. To get safe thickness, a factor of safety must be used. It is usually equal in boiler shells to 4, 5 or 6. Its value is small because the material is highly resilient and the changes of pressure are gradual, *i. e.*, there are no shocks.

This takes no account of the riveted joint which is the weakest longitudinal section; e times as strong as the solid plate; e being the joint *efficiency*, $= .75$ if the joint be double riveted. The formula then becomes $t = \frac{f\,d\,p}{2\,T\,e}$.

ubstituting values $t = \frac{6 \times 48 \times 100}{2 \times 60000 \times .75} = .32''$, say $\frac{5}{16}''$.
'he circular joints will be single riveted and joint :iency will $= .50$. But the stress is only one half as ıt as in the longitudinal joint and therefore it is stronger in proportion $(.50 \times 2)$ to $.75 = 1$ to $.75$. From this it is seen : a circular joint whose efficiency is .50 is as strong as solid plate in a longitudinal section.
'rom the value of *t* the joints may now be designed.
›iameter of rivet $= d = 1.2\sqrt{t} = 1.2\sqrt{.3125} = .672''$, say $7'' = \frac{11}{16}''$
'he pitch for a single riveted joint,

$$= p = \frac{.7854\, d^2 S + T t d}{T t}$$

ꞁut $d = \frac{11}{16}'' = .687''$

$S = 50000$ for steel

$T = 60000$ for steel

$t = \frac{5}{16} = .3125$

ubstituting these values $p = 1.42''$ say $1.5''$
'or double riveted joint

$= \frac{1.57\, d^2 S + T t d}{T t} =$ (substituting as above) $2.66''$, say
$''$.
'he margin $= d = .687'' = \frac{11}{16}''$
'he longitudinal joint will be staggered riveted and the :ance between rows $= 1.88\, d = 1.29'' =$ say $1\frac{5}{16}''$.
'he total lap in the longitudinal joint $= 4.88\, d = 3.35''$
'he total lap in the circular joint $= 3\, d = 2\frac{1}{16}''$
'he joints are therefore completely determined, and a ail of each, giving dimensions may be drawn for the use the workmen who make the templets and lay out the ets.
3. Bolts and screws as machine fastenings.
'lassification may be made as follows: 1st. *Bolts*; 2d. *:ds*; 3d. *Cap screws or tap bolts*; 4th. *Set screws*; 5th. *:chine screws.*

A bolt consists of a head and round body on which a thread is cut and upon which a nut is screwed.

When a bolt is used to connect machine parts, a hole the size of the body of the bolt is drilled entirely through both parts, the bolt is put through and the nut screwed down upon the washer. See Fig. 64.

A stud is a piece of round metal with a thread cut upon each end. One end is screwed into a tapped hole in some part of a machine, and the piece to be held against it, having a hole the size of the body of the stud, is put on, and a nut is screwed upon the other end of the stud against the piece to be held. See Fig. 65.

A cap screw is a substitute for a stud and consists of a head and body on which a thread is cut. See Fig. 66. The screw is passed through the removable part and screwed into a tapped hole in the part to which it is attached. It will be seen that a cap screw is a stud with a head substituted for the nut.

In machine designing a hole should never be tapped into a cast-iron machine part when it can be avoided. This is because cast-iron is not good material for the thread of a nut since it is weak and brittle and tends to crumble. In very many cases, however, it is absolutely necessary to tap into cast iron. It is then better to use studs in case the attached part needs to be removed often, because studs are put in once for all, and the cast-iron thread would be worn out eventually if cap screws were used. In case one machine part surrounds another, as a pulley hub surrounds a shaft, relative motion of the two is often prevented by means of a *set screw* which is a threaded body with a small square head. See Fig. 67. The end is either pointed as in Fig. 67 (b) or cupped as in (c) and is forced against the inner part by screwing through a tapped hole in the outer part.

The name machine screws covers many forms of small screws usually with screw driver heads.

All of the kinds given in this classification are made in great variety of size, form, length, etc.

Stress may come upon a bolt or stud due to screwing up which may equal or very greatly exceed the working stress. This is especially true when the nuts are screwed up to keep a joint steam tight.

Suppose that a nut is screwed up with a stress p and that a gradually increasing working stress p^1 is applied. If the bolt does not yield at all the total stress is equal to $p+p^1$. But if the addition of p^1 causes the bolt to stretch, this stretching tends to relieve the stress p and therefore the total stress upon the bolt equals some less value than $p+p^1$. If p^1 be sufficiently increased the bolt will be stretched enough so that p will be reduced to zero, and then the total stress upon the bolt would equal p^1. In case of joints that must be kept steam tight this would not be allowable because when $p = 0$ the joint would leak.

Whether a bolt yields appreciably or not under a given stress depends upon its length and cross sectional area. Reducing the latter and increasing the former tends to increase the yielding. Short studs of full cross section are therefore best for making steam joints. Longer ones, however, may be used where shocks and exceptional stress are possible and the increased yielding of the longer stud or bolt may diminish the maximum stress by causing it to act through a greater space. This same result may be accomplished by turning down the body of the bolt to the diameter of the bottom of the thread thereby increasing the length of the yielding part. See Fig. 68. The same result may be attained by drilling a hole in the bolt as in Fig. 69.

Experience has shown that the reduced bolt will endure shocks that would break the full sized bolt on a section through the bottom of the thread.

It is required to design the fastenings to hold on the steam chest cover of a steam engine. The opening to be covered is rectangular, 10″x12″. The maximum steam pressure is 100 lbs. per square inch. The joint must be held steam tight and therefore short unyielding fastenings are required,

and studs will be selected. They will be made of machinery steel of 60000 lbs. tensile strength, and will be ¾″ outside diameter because smaller studs are liable to rupture by the force applied through the wrench in screwing up. It will be assumed that the stress on the studs is equal to $p + p^1$, *i. e.*, the stress due to screwing up, plus the working stress. The diameter of a ¾″ stud at the bottom of the thread is .62″, and the area is $= .62^2 \times \pi \div 4 = .3$ square inch. The ultimate strength of the stud is, therefore, $.3 \times 60000 = 18000$. The factor of safety may be 4 because in this case the stress member is of resilient material and is not subject to shocks. Then the allowable stress on each stud would be equal to $18000 \div 4 = 4500$. If the stress due to screwing up be determined, and subtracted from the total allowable stress, the remainder will be the allowing working stress that may be applied to the stud.

The stress due to screwing up may be found as follows: a force P is applied on a wrench handle at a distance l from the centre of the stud. This acting force is opposed, just at the time of tightening the nut, by the frictional resistance between the nut and the screw and washer, and by the axial tension in the stud $= T$, which is to be found. Consider a complete revolution of the nut; the space acted through by the force $P = 2 l \pi$; by the frictional resistance $= 2 r \pi$ (r being the assumed mean radius at which the friction acts), the space acted through by $T = p =$ the pitch of the thread on the stud, The *ratio* of these spaces is the same for a small angular movement as for a complete revolution and so the equation for equilibrium when the nut is just tightened is,

$$2 P l \pi = 2 T f r \pi + T p,$$

$$\text{and } T = \frac{2 P l \pi}{2 f r \pi + p}$$

$f =$ the coefficient of friction $= .15$ for unlubricated smooth metallic surfaces.

$r =$ the mean radius of resistance of friction $= .5''$ (assumed).

l = the acting length of a wrench handle for a ¾″ nut = 9″.
p = the pitch of the thread of a ¾″ stud = .1″
P = the 30 lbs. maximum (assumed).
Substituting these values,

$$T = \frac{2 \times 30 \times 9 \times \pi}{2 \times .15 \times .5 \times \pi + .1''} = \frac{1690}{.57} = 2965.$$

The allowable stress on each stud is 4500 and the stress T, due to screwing up = 2965 for the assumed conditions, and therefore the allowable working stress on each stud = 4500 − 2965 = 1535 lbs.

The total working stress that can come on the cover equals the square inches of area in the opening covered, multiplied by the maximum working pressure per square inch = 10 × 12 = 120 square inches multiplied by 100 lbs. per square inch = 12000 lbs This divided by the allowable working pressure for each stud gives the number of studs required for strength

$$= \frac{12000}{1535} = 7.8 +.$$

Therefore 8 studs will serve for strength. But in order to make a steam tight joint, with a reasonable thickness of steam chest cover, the distance between the stud centres should not be greater than 4½″. (See Fig. 70.) The opening is 10″ x 12″ as shown and there must be a band about ⅝″ wide all around this for making the joint, and the studs must not encroach on this. This makes the distance between the vertical rows of studs 14″ and between the horizontal rows, 12″. The whole length over which the studs are to be distributed then = 12 + 12 + 14 + 14 = 52″ and if they are to be but 4.5″ apart, then the number of studs = 52 ÷ 4.5 = 11.5. From this it is seen that it is necessary to use 13 studs to make the joint tight, while 8 would serve for strength. Also, in order to get a symmetrical arrangement, it will probably be necessary to use 14 studs.

[redacted]

It is required to design proper fastenings for holding on the cap of a connecting rod like that shown in Fig. 71. These fastenings are required to sustain shocks and may be subjected to a maximum accidental stress of 40000 lbs. There are two fastenings and therefore each must be capable of sustaining safelv a stress of 20000 lbs. They should be so designed that they shall yield as much as is consistent with strength. In other words they should be made tensile springs to cushion the shocks and thereby reduce the resulting force that they have to sustain. Bolts should therefore be used, and the weakest section should be made as long as possible. Steel will be used whose tensile strength is 60000 lbs. per square inch. The stress given is the maximum accidental stress and is many times the working stress. It is therefore necessary to give the bolts only a slight excess of area of cross section over that necessary to resist actual rupture by the accidental force. Let the factor of safety be 1.25. Then the cross sectional area of each bolt must be such that it will just sustain $20000 \times 1.25 = 25000$ lbs. and this area equals $25000 \div 60000 = .416$ square inches. This area corresponds to a diameter of .73″, and that is the diameter of a $\frac{7}{8}''$ bolt at the bottom of the thread, and therefore $\frac{7}{8}''$ bolts will be used. The cross sectional area of the body of the bolt must now be made at least as small as that at the bottom of the thread. This may be accomplished as in Fig. 68 by turning down the outside of the body, or as in Fig. 69 by drilling a hole in axis of the bolt. The latter form is stronger and stiffer to resist the torsional stress that comes on the bolt when it is screwed up, because the polar moment of inertia of the cross section is greater, and it is stronger and stiffer to resist any transverse stress that may come upon it, accidentally or otherwise, because the rectangular moment of inertia of the cross section is greater. The drilled bolts are somewhat more expensive but are probably preferable.

When bolts are subjected to constant vibration there is a

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tendency for the nuts to loosen. There are numerous devices to prevent this, but the most common way is by the use of jamb nuts. Two nuts are screwed on the bolt and the under one is set up against the surface of the part that is to be held in place, and then while this nut is held with a wrench the other nut is screwed up against it tightly. Suppose that the bolt has its axis vertical and that the nuts are screwed on the upper end. The nuts being screwed against each other the upper one has its internal screw surfaces forced against the under screw surfaces of the bolt and, if there is any lost motion, as there almost always is, there will be no contact between the upper surfaces of the screw on the bolt and the threads of the nut. Just the reverse is true of the under nut; i. e. there is no contact between the under surfaces of the threads on the bolt and the threads on the nut. Therefore no pressure that comes from the under side of the under nut can be communicated to the bolt through the under nut directly, but it must be received by the upper nut and communicated by it to the bolt since it is the upper nut alone that has contact with the under surfaces of the thread. Therefore the jamb nut which is usually made about half as thick as the other, should always be put on next to the surface of the piece that is to be held in place.

14. KEYS USED AS MACHINE FASTENINGS.

Keys are chiefly used to prevent relative motion between shafts and the parts they support as pulleys, gears, &c. Keys may be divided into parallel keys, taper keys, and feathers or splines.

In the case of a parallel key the "seat," both in the shaft and the attached part has parallel sides, and the key simply prevents relative rotary motion, and motion parallel to the axis of the shaft must be prevented by some other means; probably best by set screws which bear upon the top surface of the key as shown Fig. 72. A parallel key should fit accurately on the sides and loosely at the top and bottom.

A taper key has parallel sides and has its top and bottom surfaces tapered, and is made to fit on all four surfaces, being driven tightly "home." It prevents relative motion of any kind between the parts connected. If a key of this kind have a head as shown in Fig. 73, it is called a *draw key* because it is drawn out when necessary, by driving a wedge between the hub of the attached part and the head of the key. When a taper key has no head it is removed by driving against the point with a "key drift."

Feathers or splines are keys that prevent relative rotation, but purposely allow axial motion. They are sometimes made fast in the shaft as in Fig. 74, and there is a key "way" in the attached part that slides along *a b*.

Sometimes the feather is fastened in the hub of the attached part as shown in Fig. 75, and slides in a long key way in the shaft.

John Richards' rule for keys is (see Fig. 76,) $w = \frac{d}{4}$. t has such value that $\alpha = 30°$. This rule is deviated from somewhat as shown by the following table taken from Richards' "Manual of Machine Construction." Page 58.

w =	1	1¼	1½	1¾	2	2½	3	3½	4	5	6	7	8
d =	¼	5/16	⅜	7/16	½	⅝	¾	⅞	1	1⅛	1⅜	1½	1¾
t =	5/32	3/16	¼	9/32	5/16	⅜	7/16	½	⅝	11/16	13/16	⅞	1

When d exceeds 8″ two or more keys should be used and w may then $= d \div 16$; t being as before of such value that α shall $= 30°$. The following table for dimensions for parallel keys is also from Richards' "Manual:"

d =	1	1¼	1½	1¾	2	2½	3	3½	4
w =	5/32	7/32	9/32	11/32	13/32	15/32	17/32	9/16	11/16
t =	3/16	¼	5/16	⅜	7/16	½	9/16	⅝	¾

Also this for feathers:

d =	1¼	1½	1¾	2	2¼	2½	3	3½	4	4½
w =	¼	¼	5/16	5/16	⅜	⅜	½	9/16	9/16	⅝
t =	⅜	⅜	7/16	7/16	½	½	⅝	¾	¾	⅞

For keying hand wheels and other parts that are not subjected to very great stress, a cheap and satisfactory method is to use a round key driven into a hole drilled in the joint as in Fig. 77. If the two parts are of different material, one much harder than the other, this method should not be used, as it is almost impossible in such case to make the drill follow the joint.

The taper of keys varies from ⅛″ to ½″ to the foot.

A *cotter* is a key that is used to attach parts that are subjected to a force of tension tending to separate them. Thus piston rods are often connected to both piston and cross head in this way. Also the sections of long pump rods, etc.

The sketches show machine parts held against tension by cotters. See Fig. 78 (b). It is seen that the joint may yield by shearing the cotter at AB and CD; or by shearing *CPQ* and *ARS*; or by shearing on the surfaces *MO* and *LN*; or by tensile rupture of the rod on a horizontal section at *LM*. All of these sections should be sufficiently large to resist the maximum stress safely. The difficulty is usually to get *LM* strong enough in tension; but this may usually be occomplished by making the rod larger, or the cotter thinner and wider. It is found that taper surfaces if they be smooth and somewhat oily will just cease to stick together when the taper equals 1.5″ per foot. The taper of the rod in Fig. 78 (b) should be about this value in order that it may be removed conveniently when necessary.

Relative rotation between machine parts is also prevented sometimes by means of "shrink" and "force" fits. In the former the shaft is made larger than the hole in the part that is to be held upon it, and the metal surrounding the hole is heated, usually to low redness, and because of the expansion it may be put on the shaft and on cooling it shrinks and "grips" the shaft. A key is sometimes used in addition to this.

Force fits are made in the same way except that they are put together cold, either being driven together with a heavy

sledge or, better, being forced together by hydraulic pressure. The necessary allowance for forcing, *i. e.* the excess of shaft diameter over the diameter of the hole, is given in the following table :

Diameter of Shaft, inches,	1	2	3	4	5	6	7	8	9	10
Allowance for Forcing, inches	.004	.005	.006	.006	.007	.008	.008	.009	.01	.01

Experience shows that with this allowance a steel shaft may be forced into a hole in cast iron by a total pressure of from 40 to 90 tons. There is no need of keying when parts are put together in this way.

15. On the designing of Belt Gearing. A belt must be stretched over pulleys with considerable tension in order that it shall drive, and there is therefore tension in both sides of the belt A and B, Fig. 79. This tension is the same in both sides as long as the pulleys are at rest. When, however, an increasing moment is applied to the driver tending to produce rotation, the tension in the side B is increased and in the side A is decreased till the difference of tension in the two sides is equal to the force P of the resistance to be overcome at the surface of the follower, and then rotation begins and continues as long as there is equality of driving and resisting moment. If S_n be the tension in the tight side of belt and S_0 be the tension in the slack side, then $S_n - S_0 = P$. For certain conditions of arc of contact and friction between the pulley and belt surfaces there is always a certain ratio of S_n to S_0 when slipping is impending and this ratio is expressed by the formula (See Church's Mechanics, page 183),

$$\frac{S_n}{S_0} = e^{f\alpha}$$

In which e = the neperian base = 2.718+, and f is the coefficient of friction of the belt on the pulley, and α is the π measure of the least angle of contact of the belt with either of the pulleys. Taking the common log. of this equation

$$\log \frac{S_n}{S_0} = .4343\ f\alpha$$

We have, therefore, two equations; one for the difference of S_n and S_o, and one for the ratio of S_n to S_o. Therefore if the values of P, f and a be known the equations may be combined and the values of S_n and S_o may be found. Since S_n is the maximum stress that comes on the belt, then if the cross section of the belt be made of such area that it will sustain this stress safely, the belt is properly designed. The value of P may almost always be determined from the power to be transmitted. Thus if a certain number of foot pounds A, require to be transmitted per minute, and the velocity of the rim of the pulley that receives this power is a certain number of feet per minute $= s$, then the force P will equal to the work divided by the space through which it acts, or $P = A \div s$. The value of a is easily found from the diameters of the pulleys and their distance between centres, and may usually be estimated closely enough. The value of f, the coefficient of friction, varies with the kind of belting, the material and character of surface of the pulley, and also with the rate of slip of the belt on the pulley. Some experiments made at the laboratory of the Massachusetts Institute of Technology, under the direction of Professor Lanza show that for leather belting running on turned cast iron pulleys, the rate of slip for efficient driving is from 3 to 12 feet per minute; and also that the coefficient of friction corresponding to this rate of slip is about .3. If therefore this value of f be used the slip will be kept within the above limits and the driving of the belt so designed will be satisfactory.

Table giving values of $\frac{S_n}{S_o}$ for $f = .3$ and for different values of a = least arc of contact of belt with either pulley.

a degrees	a π measure	$\frac{S_n}{S_o}$
180	3.14	2.56
175	3.05	2.49
170	2.96	2.42

165	2.87	2.36
160	2.78	2.30
155	2.69	2.24
150	2.61	2.18
145	2.52	2.12
140	2.43	2.07
135	2.35	2.02
130	2.26	1.96
125	2.17	1.91
120	2.08	1.86
115	2.00	1.82
110	1.91	1.77
105	1.83	1.73
100	1.74	1.68
95	1.65	1.64
90	1.56	1.59

Problem. A single acting pump has a plunger that is 8″ = .666 feet in diameter and its stroke has a constant length of 10″ = .833 feet. The number of strokes per minute is 50. The plunger is actuated by a crank and the crank shaft is connected by spur gears to a pulley shaft, the ratio of the gears being such that the pulley shaft runs 300 revolutions per minute, and the pulley that receives the power from the line shaft is 18″ in diameter. The pressure in the delivery pipe is 100 lbs. per square inch. The axis of the line shaft is at a distance of 12 ft. from the axis of the pulley shaft, and runs 150 revolutions per minute. Since the line shaft runs half as fast as the pulley shaft, the pulley on the line shaft must be twice as large in diameter as that on the pulley shaft, or 36″.

The work to be done per minute, neglecting the friction in the machine is equal to the number of pounds of water pumped per minute multiplied by the head in feet against which it is pumped. The number of cubic feet of water per minute equals the displacement of the plunger in cubic feet multiplied by the number of strokes per minute = $\frac{.666 \times \pi}{4}$

$\times .833 \times 50 = 14.5$. One cubic foot of water weighs 62.4 lbs. and therefore the number of pounds of water pumped per minute $= 14.5 \times 62.4 = 907$.

One foot vertical height or "head" of water = a pressure of .435 lbs. per square inch; and therefore 100 lbs. per square inch corresponds to a "head" of $100 \div .435 = 230$ ft.

The work done per minute in pumping the water therefore is equal 907 lbs $\times$ 230 ft. = 208610 ft. lbs.

The velocity of the rim of the belt pulley $= 300 \times 1.5\pi =$ 1410 feet per minute. Therefore the force $P = S_n - S_o =$ 208,610 ft. lbs. per minute $\div$ 1410 ft. per minute = 147 lbs.

To find α see Fig. 80. $\sin\theta = \frac{R-r}{l} = \frac{9''}{144} = .0625$

$$\text{Therefore } \theta = 3^\circ\ 35'$$

$$\alpha = 18^\circ - 2\theta = 180^\circ - 7^\circ\ 10'$$

$$= 173^\circ \text{ nearly.}$$

The value of $\frac{S_n}{S_o}$ in the table for $175^\circ = 2.49$ and this value may be used without important error.

Therefore we have $S_n - S_o = 147$

$$\text{and } \frac{S_n}{S_o} = 2.49$$

Combining these equations S_n is found to be equal to 245 lbs. = the maximum stress that comes on the belt. Experiment shows that 70 lbs. per inch of width of a *laced single* belt is a safe working stress. Therefore the width of the belt $= 245 \div 70 = 3.5''$. The friction of the machine might have been estimated and added to the work to be done. The work of a single acting pump like the one considered is only through a part of the stroke and therefore a suitable fly wheel would be put on the pulley shaft to equalize the tension on the belt.

Problem: A 60 horse power dynamo requires to run 1500 revolutions per minute, and has a 15″ pulley on the shaft. Power is supplied by a line shaft running 150 revolutions per minute. A suitable belt connection is to be designed. The

ratio of velocities of dynamo to line shaft is 10 to 1, and the pulley on the line shaft would have to be ten times as large as that on the dynamo or 12.5 ft. diameter, if the connection were direct. This is clearly inadmissible and therefore the increase in speed must be obtained by means of an intermediate, or counter shaft. Suppose that a pulley 48″ diameter is the largest that can be used on the counter shaft; then the necessary speed of the countershaft $= 1500 \times \frac{15}{48} =$ 470 nearly. The ratio of diameters of the required pulleys for connecting the line shaft and the counter shaft $= \frac{470}{150} = 3.13$. Suppose that a 60″ pulley can be used on the line shaft, then the diameter of the required pulley for the counter shaft will $= \frac{60}{3.13} =$ 19″ nearly. Consider first the belt to connect the dynamo to the counter shaft. The work $= 60 \times 33000 = 1980000$ ft. lbs. per minute; the rim of the dynamo pulley moves $\frac{\pi\ 15}{12} \times 1500 = 5890$ ft. per minute.

Therefore the force $P = \frac{1080000}{5890} = 336$ lbs.

Therefore $S_n - S_o = 336$ lbs.

The axis of the countershaft is 10 feet from the axis of the dynamo, and as before $\sin\theta = \frac{R-r}{l} = \frac{24-7.5}{120} = .1378$

$$\text{Therefore } \theta = 8^\circ \text{ nearly}$$

$$\alpha = 180 - 2\,\theta = 164^\circ$$

$$\text{From table } \frac{S_n}{S_o} = 2.36.$$

From these equations $S_n = 583$ lbs. and the safe width of single belting $= 583 \div 70 = 8.34''$, say 8.5″.

The width of the belt to connect the line shaft to the countershaft may be found by the same method.

If belts run in a horizontal or inclined position, they should be so arranged that the slack side shall be the upper

side, so that the effect of the sag of the slack side due to gravity, shall be to increase the arc of contact, instead of to decrease it, as would be the case if the slack side were the lower side. Increasing the arc of contact increases the ratio of S_n to S_0 and so gives more efficient driving for any given tightness of belt.

In the use of belting, if it is necessary to put one shaft vertically above the other the smaller pulley should be on the upper shaft if possible, because the weight of the belt tends to increase the normal pressure between the belt and the upper pulley, and to decrease it in case of the lower pulley. This increased normal pressure means increased friction, and therefore increased "grip" of the belt on the pulley. The smaller pulley has a less arc of contact, and hence needs the increased friction even at the expense of decreased friction in the larger pulley.

In the tranmission of power by belting care should be taken that the distance between the shafts carrying the pulleys be not too small; especially if there is the possibility of sudden changes of load. Belts have some elasticity and the total yielding under any given stress is proportional to the length, the area of cross section being the same. Therefore a long belt becomes a yielding part, or spring, and its yielding may reduce the stress due to a suddenly applied load to a safe value, whereas in the case of a short belt, with other conditions exactly the same, the stress, because of the very much less yielding, might be sufficieut to rupture or weaken the joint.

A given velocity ratio may be transmitted between two shafts by an infinite number of pairs of pulleys. Thus in Fig. 81 a velocity ratio of 1 may be transmitted from O to O′ by the pulleys A and B or by A′ and B′. Belting always has some stiffness to resist flexure. In the first case the belt has to be continuously bent into the curve of the circumference of A and B, and in the second case that of A′ and B′. Energy is required in either case; energy which is used up

in overcoming the friction of the fibres of the belt on each other ; but the energy is greater where the curvature is greater, and that is with the smaller pulleys. So it is seen that in any given case as large pulleys as is practicable should be used. This shows also one reason why narrow belts running at high velocities on pulleys of large diameter are more efficient transmission members than wide belts that run slowly on pulleys of small diameter, and explains the tendency of modern practice toward the former. It also explains the greater efficiency of the various forms of link belts that yield so readily to a flexure stress.

In Fig. 81 suppose that the belt A′B′ runs toward B′. On reaching B′ any part of the belt resists having its path changed from a straight line to a circle. In other words the centrifugal force of the belt tends to carry it out from the pulley as shown by the dotted line. At low belt velocities this tendency has little effect ; but at high velocities the effect is very decided, and since it reduces the arc of contact it necessarily reduces the capacity of the belt for driving. Practical experience shows that this tendency puts a practical limit to the increase of belt velocities at about 5000 ft., or 1 mile per minute. This applies especially to cases in which there is a great difference in the velocity of the shafts connected, and therefore a great difference in the size of the pulleys, because the smaller pulley has necessarily a small arc of contact, and this must not be decreased by the centrifugal tendency of the belt to leave the pulley.

For figuring approximate belt capacity the following method will serve. A single belt, 1 inch wide, running 600 feet per minute, will transmit a horse power. Suppose that a belt is to run 2400 feet per minute and that it is required to transmit 30 horse power ; how wide should it be ? 1 inch of width at 600 ft. will transmit 1 HP, and therefore 1 inch of width at 2400 ft. will transmit 4 HP, and the necessary width of belt for transmitting 30 HP equals $30 \div 4 = 7.5''$. The constant 600 above is sometimes made as large as 1000 and sometimes as small as 400.

16. The designing of Spur and Bevel Gears.—The tendency in modern machine practice is to leave the design and construction of gear wheels to those who make a specialty of their manufactnre. The expense of many accurately made gear patterns, and of machines for the accurate cutting of gears is so great that economy forbids the investment to those who use comparatively few gears. If, however, a manufacturer can make a specialty of gears and get orders enough to keep his patters and gear cutters in use almost constantly, then the investment begins to yield such a profit that the manufacturer can invest money in the improvement of the foundry and machine shop methods for the production of gears. To this is due the wonderful accuracy and smooth running of modern gear wheels. In most cases therefore today the designer who needs to use gear wheels in the machine he is designing finds in the gear list of some manufacturer, the gears that suit his needs, and orders them. Or else the gears are designed and the "blanks" turned up and sent to the gear manufacturer to be cut.

This is simply one example of the tendency, which is well nigh universal, for an industry as it develops and becomes more complicated in its details, to break up into specialties. The result being almost invariably an increase in the excellence of the product.

This does not, however, excuse the designer from the obligation to understand methods of gear design, for he must be able to specify to the manufacturer of gears just what is needed to fulfill the special requirements of the case in hand. There also sometimes arise cases that require special gears that are not found on the lists of any manufacturer, and he must be able to meet these emergencies. The following problems are given not in any hope of covering the entire ground, but as suggestions as to the method of work.

Problem.—It is required to transmit 45 HP from a shaft 3″ in diameter that runs 100 revolutions per minute, to a parallel shaft whose axis is 5 ft. distant and which is required to run 300 revolutions per minute. Spur gears are to be de-

signed that shall accomplish this result satisfactorily. (See Fig. 82.) Locate the centres O and O′ 3 ft. apart. The velocity ratio is 3 to 1, and therefore the ratio of the pitch radii will be 1 to 3. The line of centres is to be divided into parts that are to each other as 1 is to 3. Divide OO′ into 4 parts and lay off one of these parts from O toward O′ on the line of centres. Then OP is to O′ P as 1 is to 3 and therefore P is the pitch point, and circles drawn about O and O′ through P will be the pitch circles of the required gears, and their diameters are 18″ and 54″. The velocity of the pitch surfaces $= \frac{54\pi}{12} \times 100 = \frac{18\pi}{12} \times 300 = 1413$ ft. per minute. The work to be transmitted $= 45$ HP $= 45 \times 33000 = 1485000$ foot pounds per minute. The force at the pitch surface therefore $= \frac{1485000}{1413} = 1051$ lbs. The circular pitch will be assumed and $= 2''$ the corresponding width of face for the requisite strength will be determined.

(See Fig. 83.) Assume that the tooth is a cantilever whose h equals the thickness of tooth on the pitch line, and whose b is the width of face of the gear to be found. Assume that the force is applied at the extreme point of the tooth and that one tooth has to withstand the entire force; *i. e.*, that only one pair of teeth are in contact at a time. It is necessary now to determine a proper factor of safety. There is certainty that the teeth will be required to resist shocks, because if there is no backlash to begin with, there will be after the gears have worn for a long time, and any sudden variation of load will cause the teeth to strike blows on the teeth with which they engage. The gears will be made of cast iron and so an unresilient material will be required to resist shocks, and a factor of safety is required for this reason. There should also be a factor of safety to guard against failure because of shrinkage stresses and spongy material in the casting. For these reasons 10 will be used for the factor of safety, and the tooth must be made to resist a stress 10 times as great as the working stress, $= 1051 \times 10 = 10510$ lbs.

From Church's Mechanics, page 249.

$M = Pl$ for a cantilever $= \frac{pI}{e} = \frac{p\,b\,h^2}{6}$

Therefore $b = \frac{6\ Pl}{p\,h^2}$

For the problem in hand

b = required width of face of gear.

P = the total stress that the tooth is to be capable of resisting.

l = the total depth of space between the teeth = 1.37 for 2″ pitch from table of proportions of gear teeth.

p = unit ultimate tensile strength of cast iron in tension = 18000.

h = thickness of tooth on the pitch line = 1″ for 2″ circular pitch.

Substituting, $b = \frac{6 \times 10{,}510 \times 1.37}{18{,}000 \times 1} = 4.8''$

Therefore having assumed a circular pitch = 2 the necessary width of face to meet the assumed conditions is 4.8″. By comparing these values with gear lists they are found to agree closely with practice. If there had been too great disagreement between the results obtained and practice, it would have been necessary to go back and assume another value for pitch and find the corresponding value for width of face and to continue this until consistent values are obtained. It will be clear that the circular pitch to meet any required conditions could be found if the width of face is limited to a given value by other conditions in the machine. If it should occur that the width of face is limited, and the value of circular pitch corresponding for a cast iron gear is absurdly large, the resource is to use a material of higher ultimate strength, as machinery steel or steel casting.

For the design of small gears when diametral pitch is used the following is necessary.

Let D = the diameter of the pitch circle.

" D' = the outside diameter of the gear.

" P = the diametral pitch.

" $N =$ the number of teeth.

" $W =$ the width of face.

Diametral pitch from definition is the number of teeth per inch of pitch diameter, and therefore

$$P = \frac{N}{D}$$

The addendum is made such that $P = \frac{N+2}{D'}$ or $D' = \frac{N+2}{P}$

An empirical formula for width of face which may be de-deviated from if necessary, is, $W = \frac{8''}{P} + \frac{1''}{4}$

In case of gears that are used for transmission of small stress, as the change of gears of a lathe,

$$W = \frac{4''}{P} + \frac{1''}{Q}$$

Problem. It is required to design a pair of change gears for a lathe for the following conditions, distance between centres $= 6''$; velocity ratio $= 2$ to 3 diametral pitch $= 10$. Divide the distance between centres into $2+3=5$ parts; 2 of these parts will equal the pitch radius of the smaller gear, and 3 parts will equal the pitch radius of the larger gear. Therefore the pitch diameters will be $4.8''$ and $7.2''$.

Let A represent the smaller gear and B the larger.

Then for A, $N = PD = 10 \times 4.8 = 48$.

Then for B, $N = \quad 10 \times 7.2 = 72$.

Outside diameter for $A = D' = \frac{N+2}{P} = \frac{48+2}{10} = 5''$

Outside diameter for $B = D' = \frac{72+2}{10} = 7.4''$

Width of face $= W = \frac{4''}{10} + \frac{1''}{2} = .9''$ say, for convenience in measurement, $1.875'' = \frac{7}{8}''$.

Problem. A turbine water wheel with a vertical axis has a maximum capacity of 40 HP, running 600 revolutions per minute. It is required to design a pair of bevel gears that shall serve to transmit the energy of this water wheel to a

horizontal shaft that is required to run 200 revolutions per minute. The velocity ratio of 200 to 600 = 1 to 3. (See Fig. [illegible].) Draw AB and CD at right angles to each other to represent the axes of rotation; draw a line parallel to CD, at a distance from it of one unit, on any assumed scale; draw a line parallel to AB, at a distance from it of three units, on the assumed scale; join O′, the intersection of these lines, and O the intersection of the axes of the shafts. Every pair of lines at right angles to the axes of rotation, whose ratio to each other is 1 to 3, will intersect in OO′. Therefore, OO′ is the line of contact of pitch cones that would, if slipping were prevented, transmit by rolling contact a velocity ratio of 1 to 3. In this case the pinion, or smaller gear, should be made as small as possible in order to limit the size of the larger gear. Lay off from AB a distance ab, equal to the radius of the water wheel shaft, equal in this case to 1.5″. Lay off bc equal to ab, for the thickness of the hub of the gear. Lay off cd, an assumed value that must be greater than the depth of the space between the teeth at the small end of the pinion below the pitch line. Through d draw a line parallal to CD. Its intersection with OO′ is the point where the pitch circle of the small end of the pinion pierces the plane of the paper. The width of face of bevel gears is usually made equal to one-third of slant height of the pitch cones. Therefore, dividing eO into two equal parts, and laying off eE, equal to one of them, gives the proper width of face. Construction in this case gives eE = 5.5″. An approximate ratio between width of face and circular pitch is given in the formula.

W = 2.5 P in which W is the width of face and P is the circular pitch. Then P = W ÷ 2.5 = 5.5 ÷ 2.5 = 2.2″. This is only an approximation and so 2″ may be taken as the circular pitch. The tooth thus determined may be checked for strength as follows: Consider that the tapering tooth of the bevel gear is equivalent to a tooth of uniform cross section, equal to the average cross section of the tapering tooth. Treat this as a cantlever to find the maximum fibre

stress for the conditions considered, and compare this with the safe value for the material used, to see if the factor of safety is sufficiently large. The dimensions of cross section of tooth, at the middle of the length, are found by measuring a scale drawing.

h = the thickness of the tooth at the pitch line = .64″.

l = the depth of space between the teeth = .9″.

b = width of face of the gear = 5.5″.

The mean force that the teeth have to resist, is equal to the foot pounds that require to be transmitted per minute, divided by the mean velocity of the pitch cone surface in feet per minute. The mean diameter of the pitch cone = 8.75″. The mean velocity will then equal $\frac{8.75'' \times \pi}{12} \times 600$ revolutions per minute = 1370 ft. per minute.

The energy to be transmitted per minute = 40 × 33000 = 1320000 foot pounds.

Therefore, $P = \frac{132000}{1379} = 960$ lbs.

$$Pl = \frac{pI}{e}$$

$$p = \frac{Ple}{I} = \frac{6Pl}{bh^2}$$

Substituting $= \frac{6 \times 960 \times .9}{5.5 \times .64^2} = 2300.$

If the ultimate strength of the cast iron of which the gears are to be made, be taken as 18000, then the factor of safety for the case considered would equal 18000 ÷ 2300 = 7.8

In this problem other factors might have been given, and then the order of solution might have been changed, or the the whole method might have been different. The method given is not given as an invariable one for the figuring of bevel gears, but simply to indicate what may be done. It will be seen that all results are necessarily approximate and are to be used only to aid the judgment of the designer.

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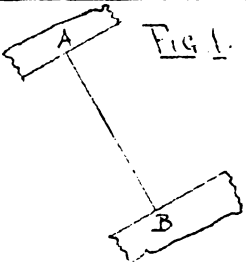
Fig 1.



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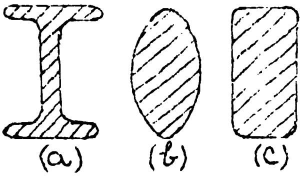

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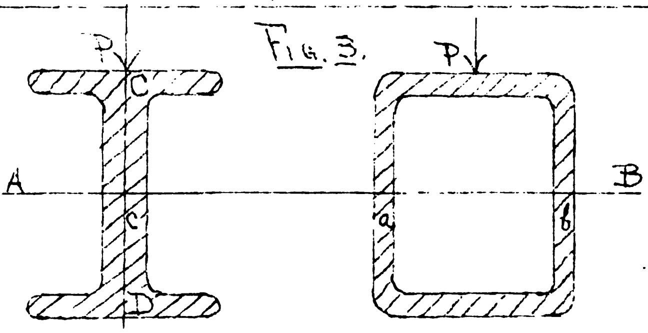


Fig 4

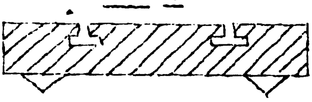
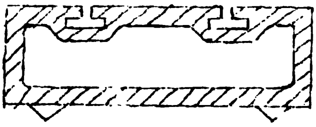


Fig 6



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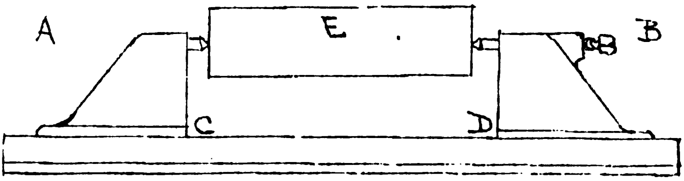


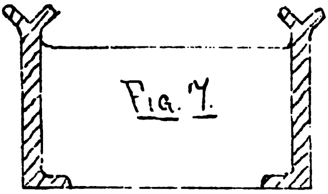
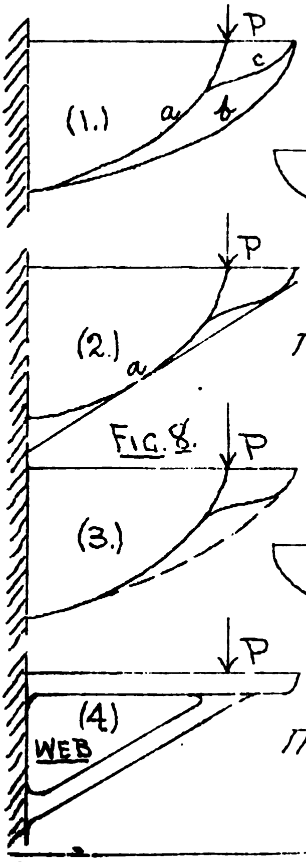


Fig. 7.



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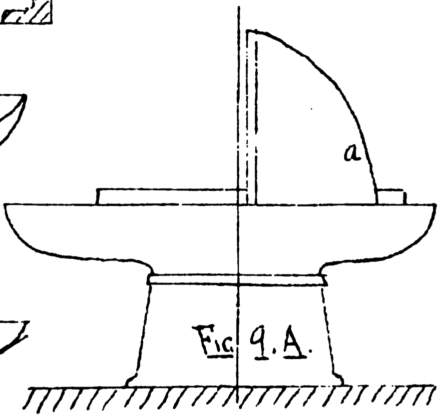

Fig. 9. A.

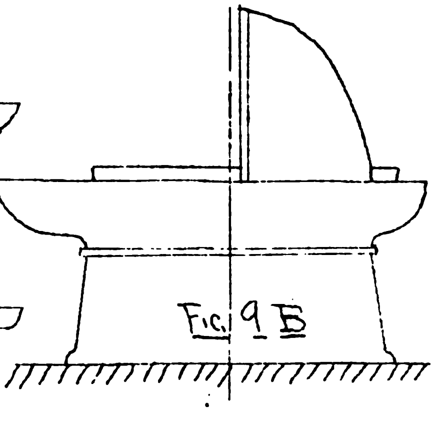

Fig. 9 B

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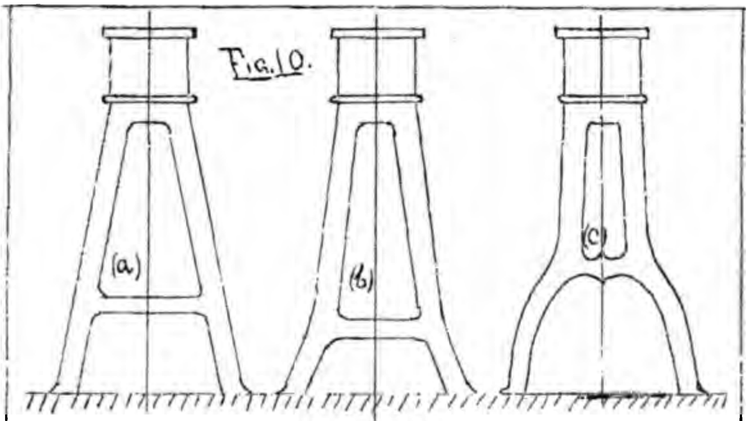

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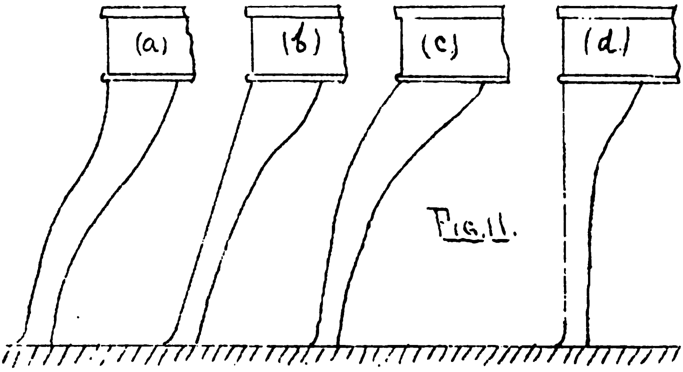

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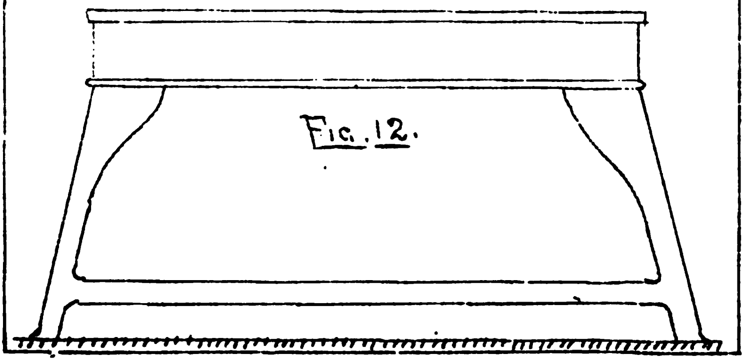
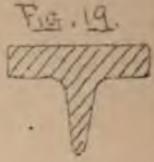


Fig. 12.

Fig. 17.

Fig. 18.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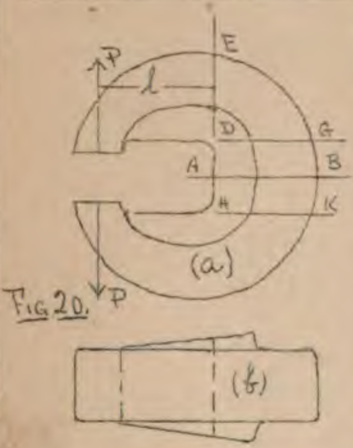
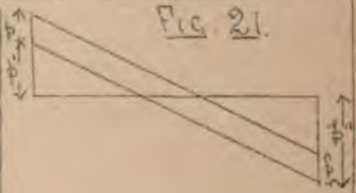


Fig. 20.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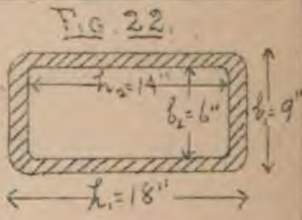
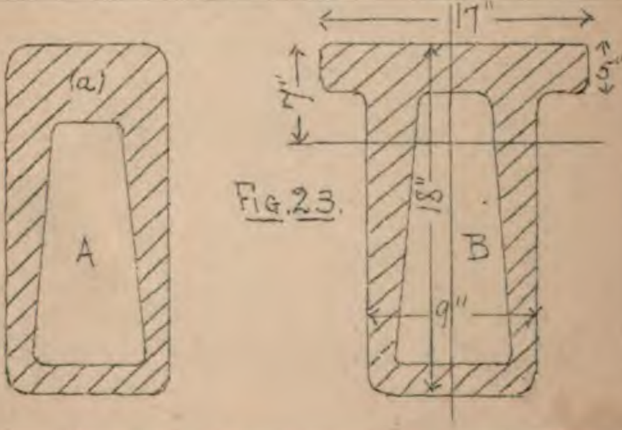


Fig. 22.



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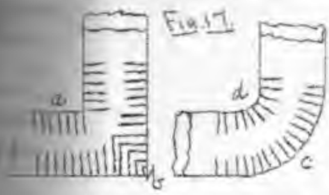
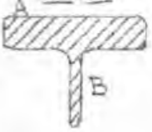


Fig. 17.



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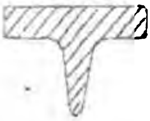
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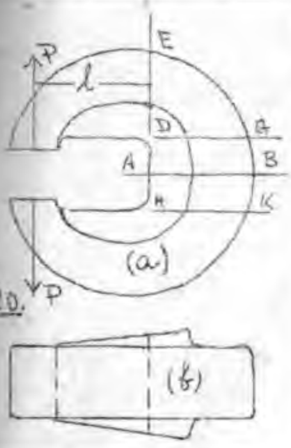

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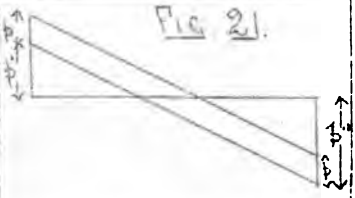
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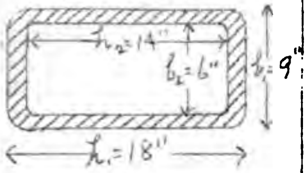

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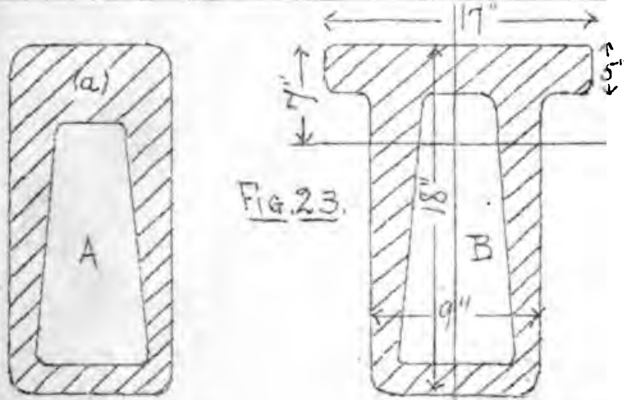

Fig. 23.

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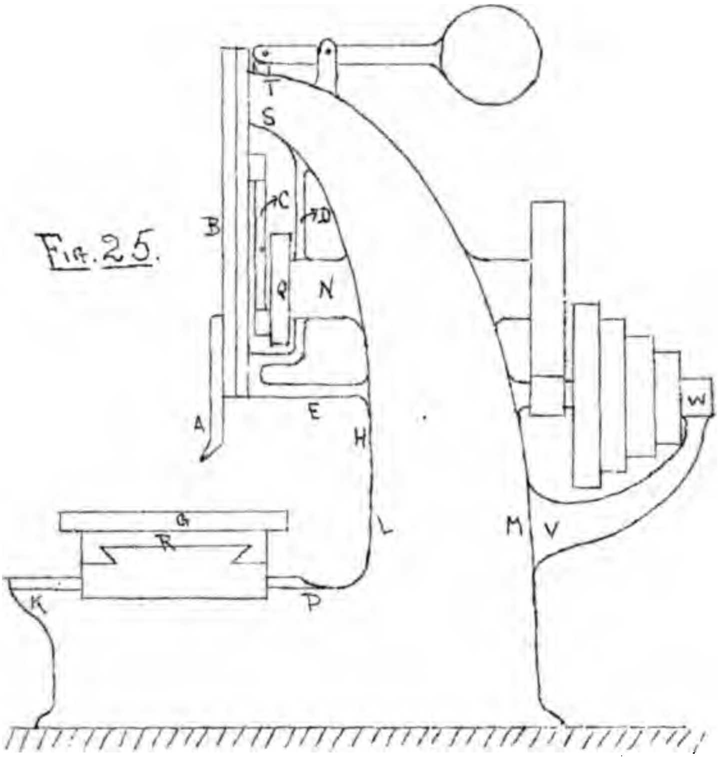

Fig. 25.

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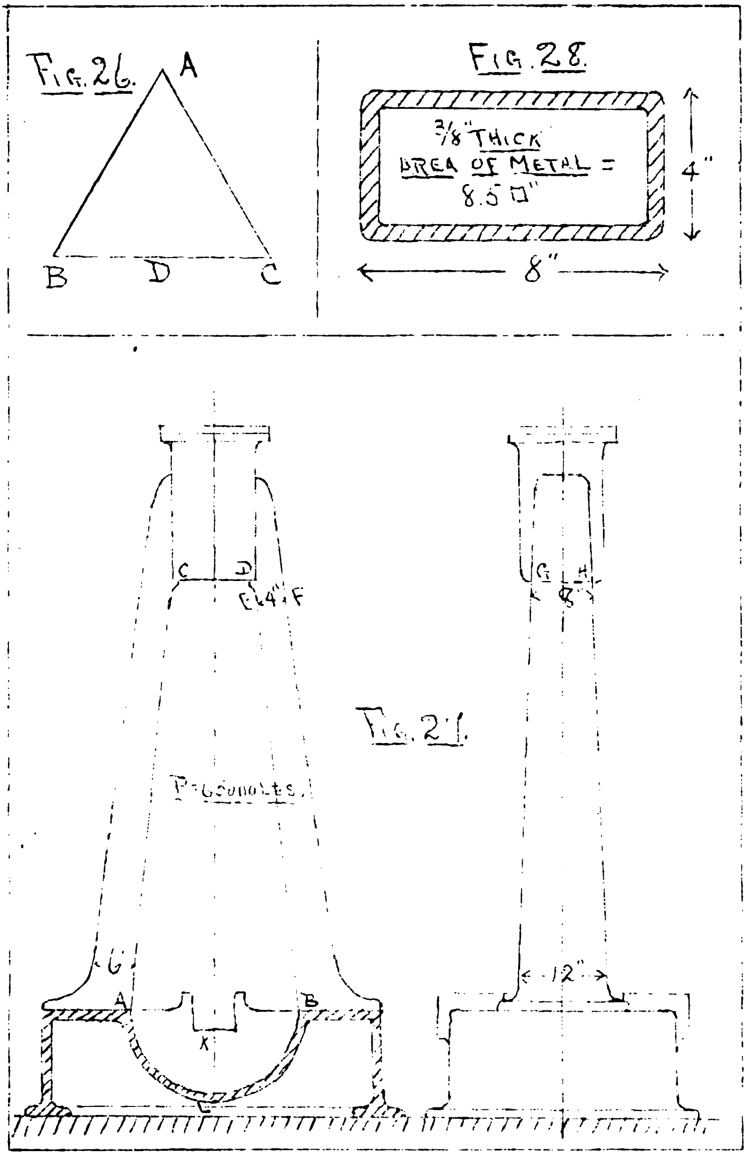

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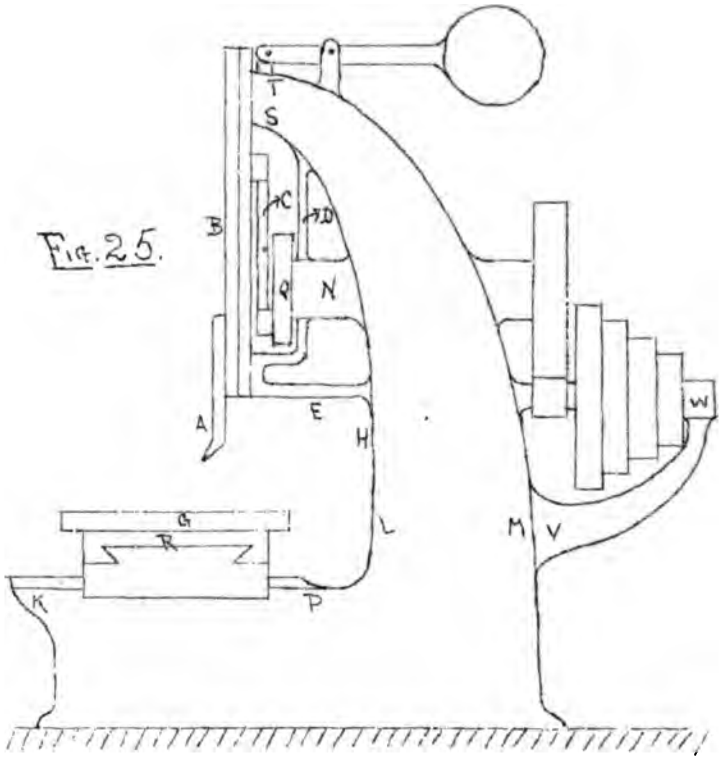

Fig. 25.

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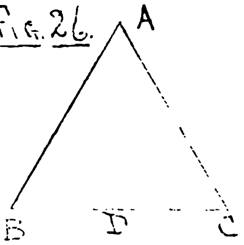
Fig. 26.



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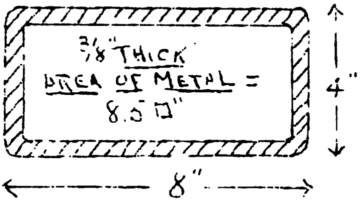

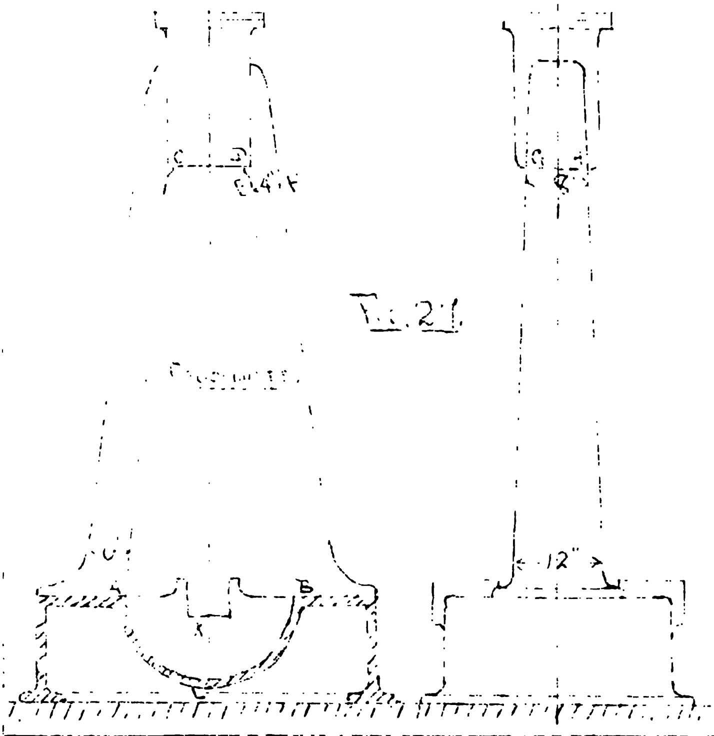

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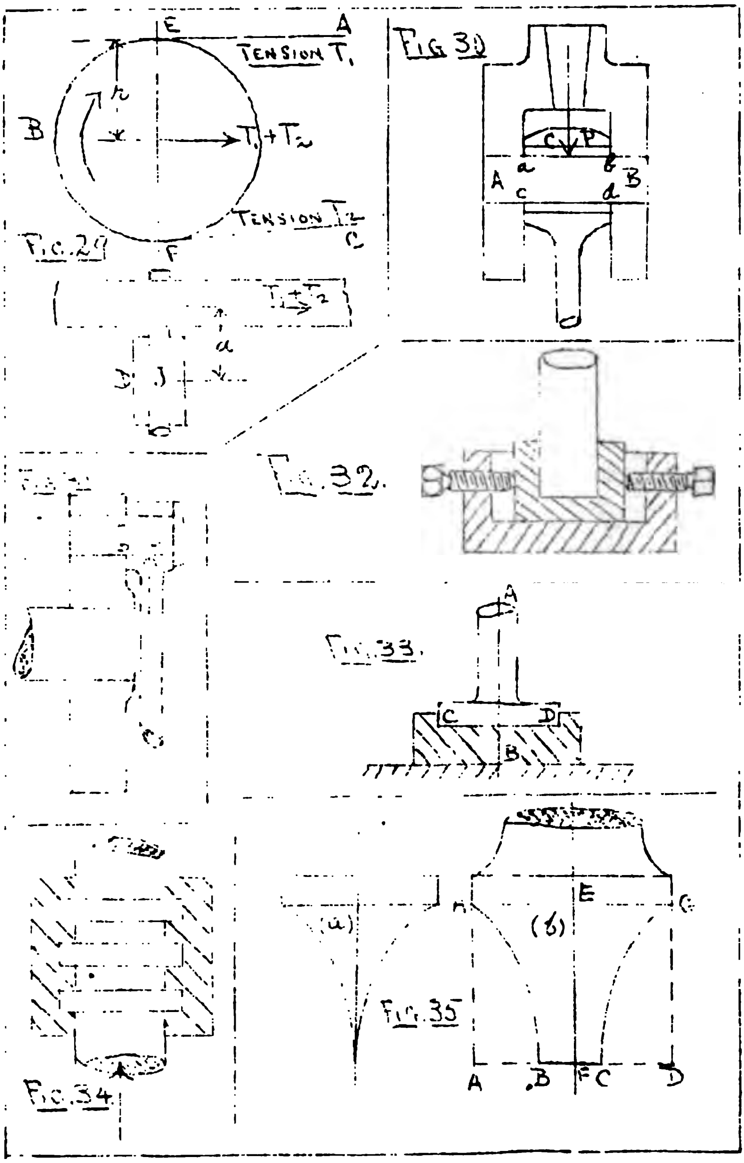
E
A
TENSION T_1
B
h
T_1+T_2
TENSION T_2
C
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FIG. 29
T_1+T_2
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FIG 30
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FIG. 32.
FIG. 33.
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FIG. 34
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(b)
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FIG. 35
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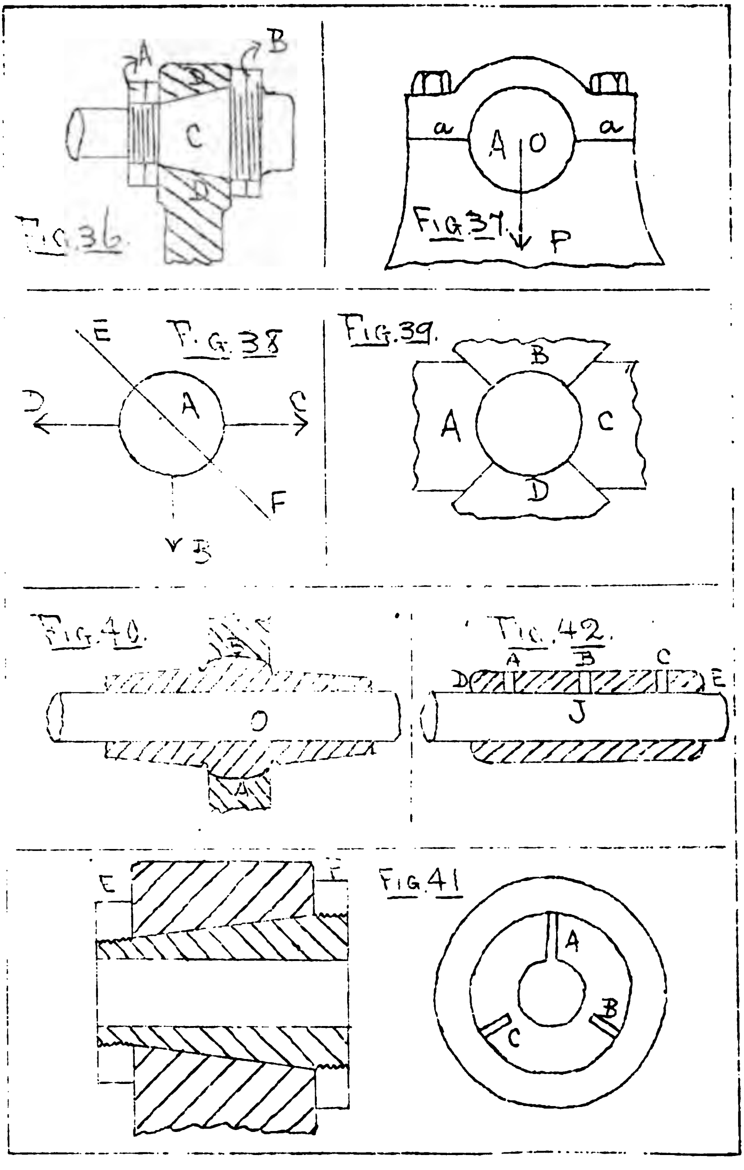

Fig. 36
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Fig. 39
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Fig. 41
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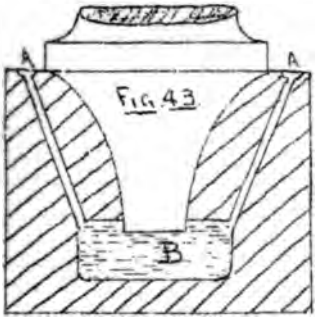

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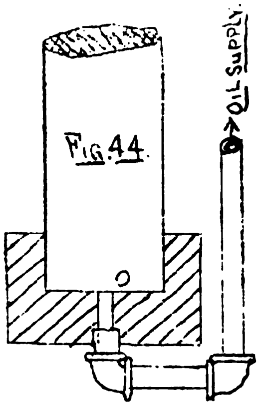

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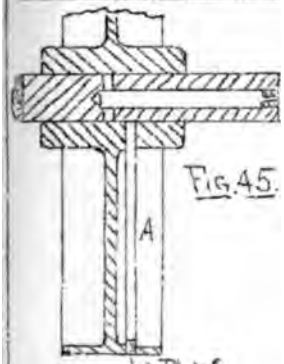

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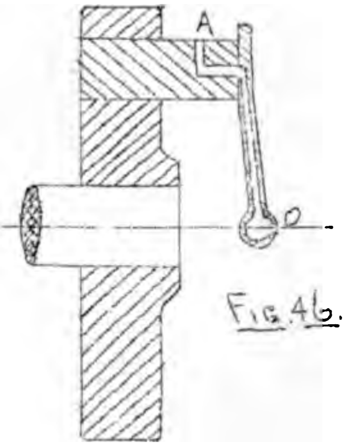
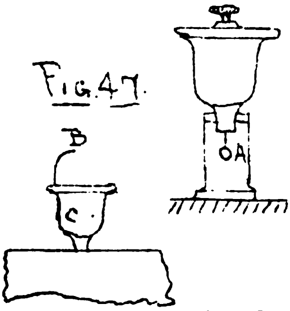
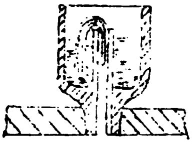


Fig. 46.

Fig. 47.





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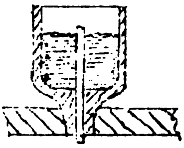

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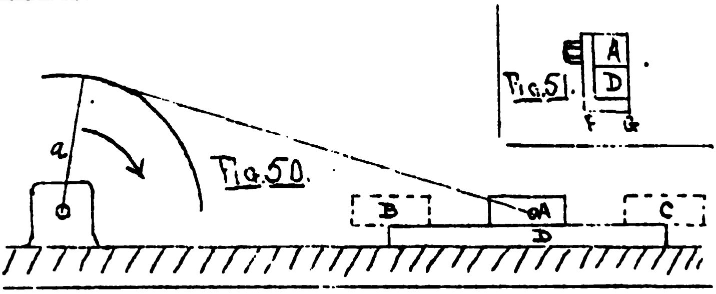


Fig. 50.

Fig. 51.

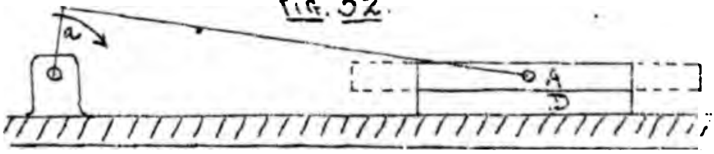


Fig. 52.

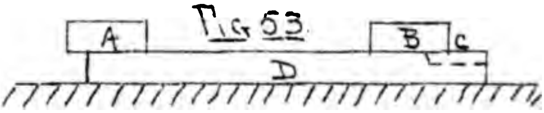


Fig. 53.

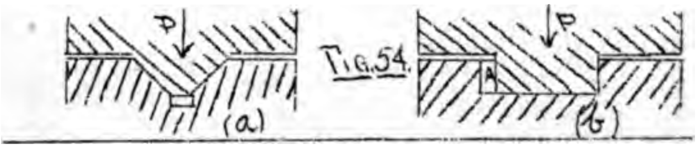


Fig. 54.

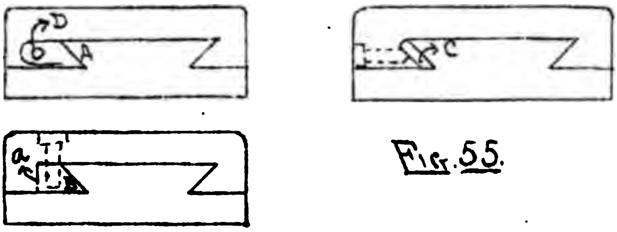


Fig. 55.

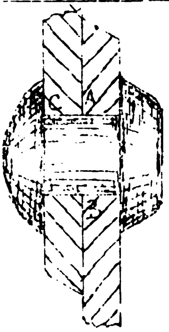


Fig. 61

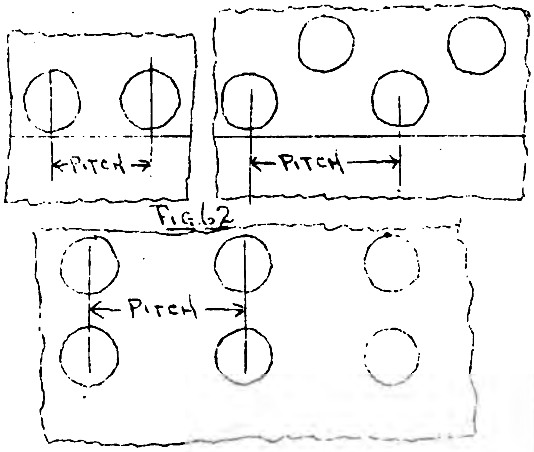


Fig. 62

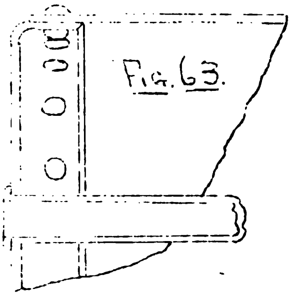


Fig. 63.

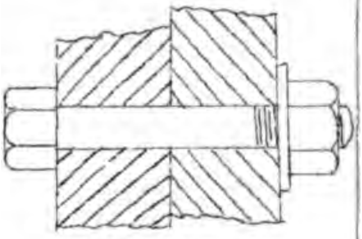


Fig. 64.

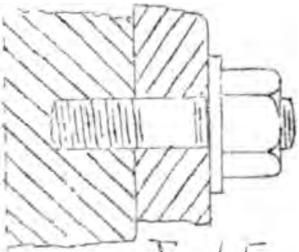


Fig. 65.

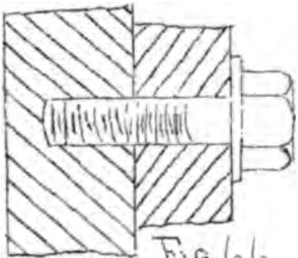
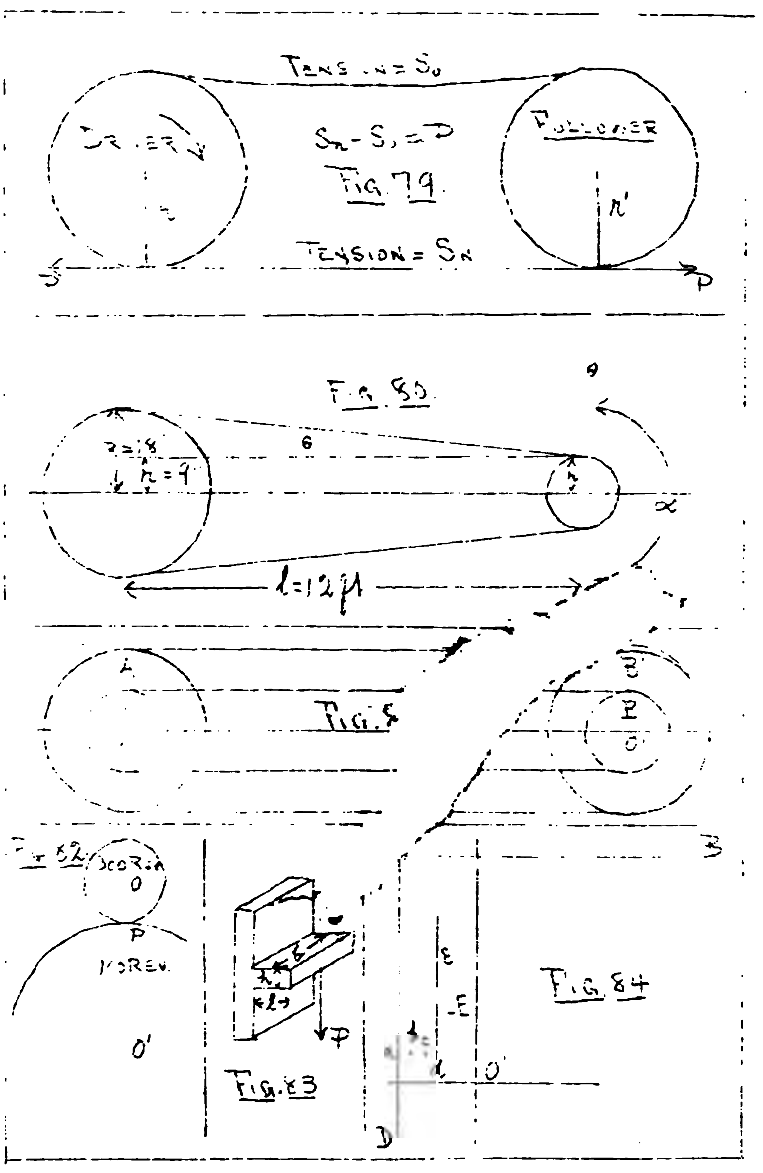


Fig. 66

[redacted]



TENSION = S
FOLLOWER
Sn - S, = P
FIG. 79.
r'
TENSION = Sn
P
FIG. 80.
l = 12 ft
α
FIG. 8
P
FIG. 82
O
P
O'
FIG. 83
P
E
O'
D
FIG. 84

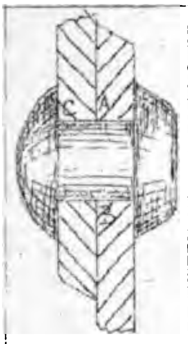


Fig. 61.

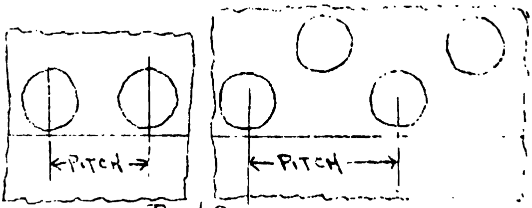
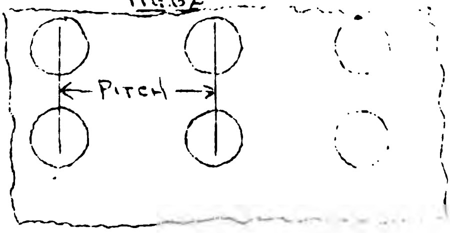


Fig. 62



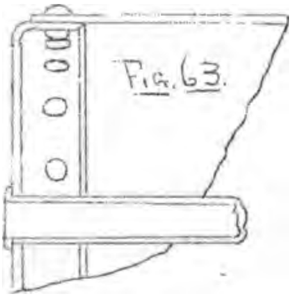


Fig. 63.

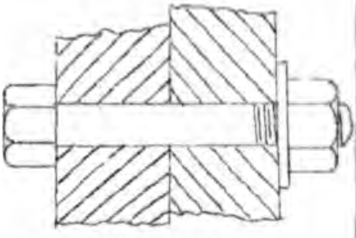


Fig. 64.

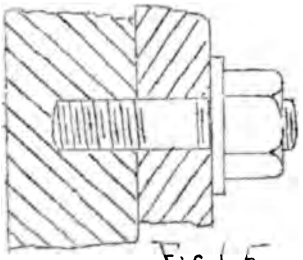


Fig. 65.

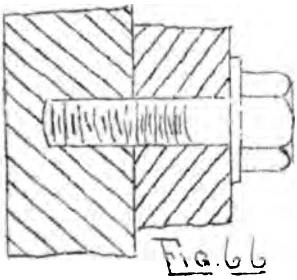
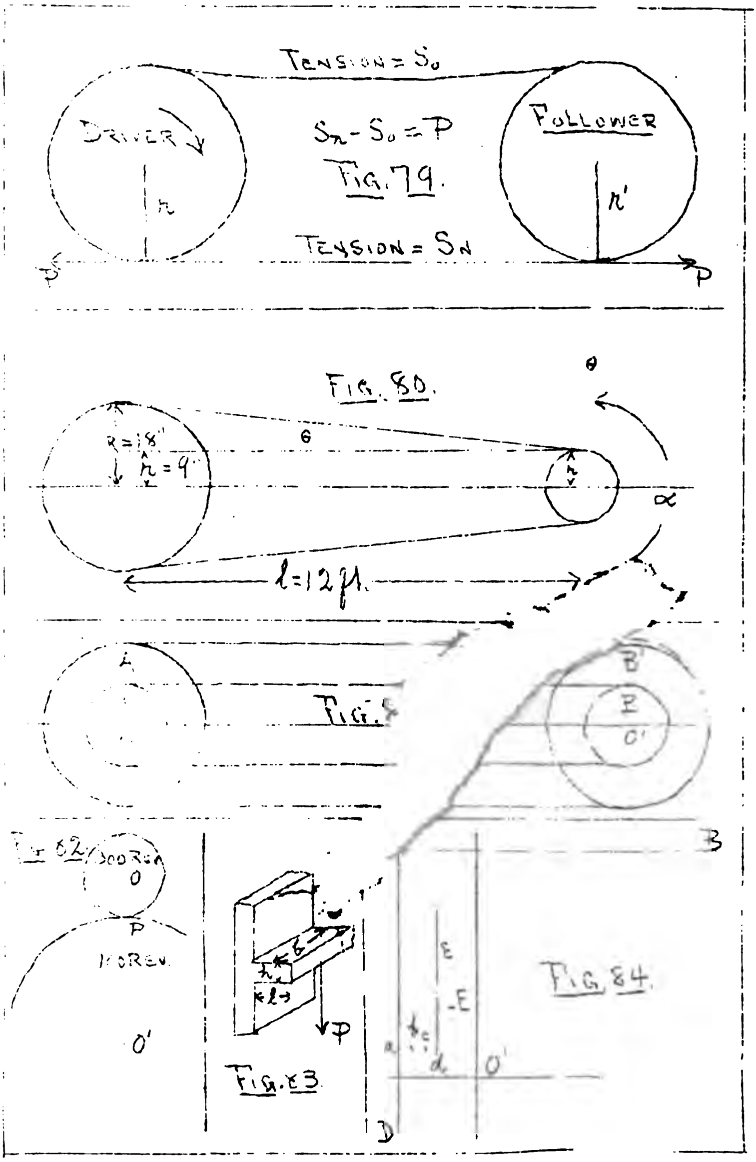


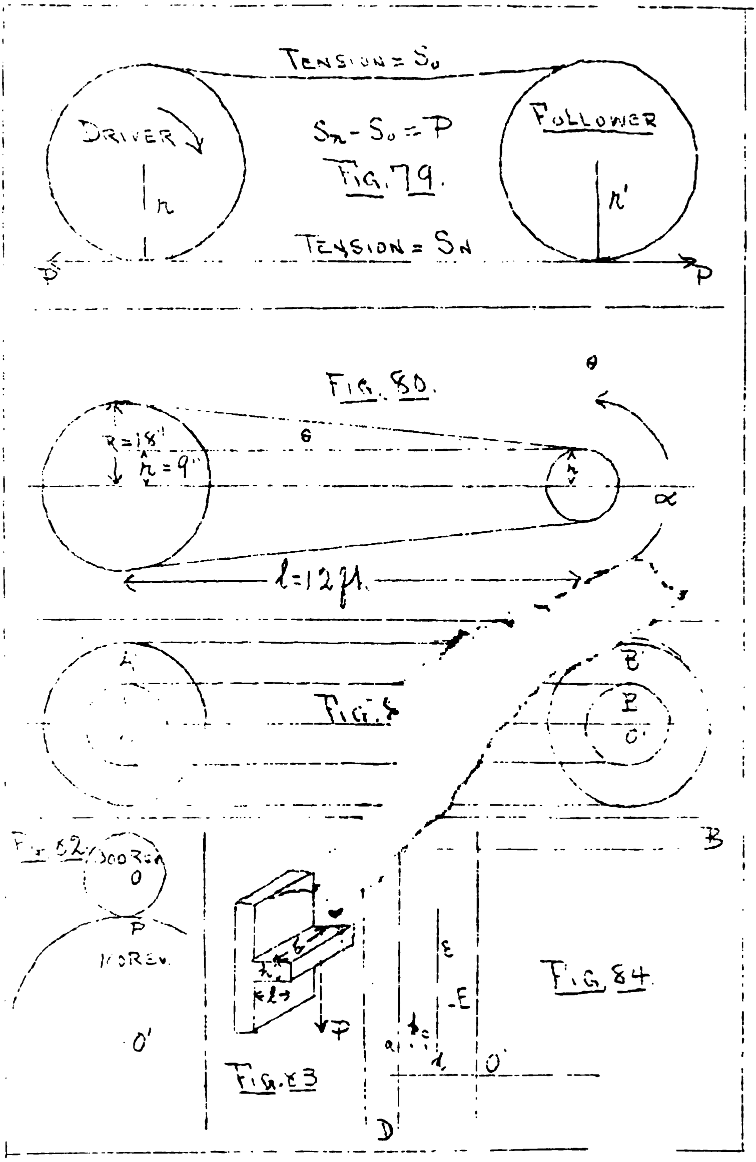
Fig. 66

[redacted]



TENSION = S_0
DRIVER
FOLLOWER
$S_n - S_0 = P$
FIG. 79.
r
r'
TENSION = S_n
P
P
FIG. 80.
θ
θ
$R = 18''$
$r = 9''$
r
α
$l = 12$ ft.
A
FIG. 8
B'
E
O'
B
FIG. 82.
300 Rev.
O
P
100 Rev.
O'
b
h
l
P
FIG. 83.
E
-E
h_c
a
d
O'
D
FIG. 84.

[redacted]


TENSION = S_0
DRIVER
FOLLOWER
$S_n - S_0 = P$
FIG. 79.
r
r'
TENSION = S_n
P
FIG. 80.
R = 18"
r = 9"
θ
α
l = 12 ft.
FIG. 81
A
B
P
C
FIG. 82
300 REV.
O
P
100 REV.
O'
FIG. 83
P
FIG. 84
E
D
O'
B

[illegible]

[illegible]

[redacted]

[redacted]